Student Beliefs about Mathematics
and
Their Effect on Academic Performance

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Consider the following statements made by several students:

Student 1: "I believe that any math problem can be solved, if enough effort is put in"

Student 2: "I don't want to memorize formulas, but I want to understand how they work"

Student 3: "I want the professor to show many examples so that I can memorize how to use the formula"

Student 4: "I am not a math person"

Student 5: "Math has been invented by clever mathematicians and is all about the rules we must follow"

Student 6: "Mathematics is all about rules; it has nothing to do with creativity"

Student 7: "I have a weak math foundation, but I am sure I can become very strong if I study hard enough"

Which of these beliefs are "good" and which are not? Why?

Major non-availing student beliefs:

"I am not a math person"

"Math is a black box, filled with mysterious rules"

"Math does not develop creativity: it is all about rigid rules"

"I don't need mathematics. Today calculators and computers can compute everything"

1. **Why are these beliefs detrimental?**

2. **How are these beliefs connected with mathematics performance?**

3. **How are beliefs formed?**

4. **Is it possible to change non-availing beliefs? If yes, which method of teaching does it best?**

WHAT DO STUDENTS BELIEVE AT SENECA?

Data from Seneca College, School of Business. A sample of 52 students from Bachelor of Commerce Degree:

I need to memorize	38%
Math is not necessary	12%
I know I will have difficulty	54%
Math develops creativity	50%
Math helps think critically	81%
Math success is in effort	46%
I do not have a math mind	16%

On average, 62% of beliefs where availing. The proportion of those who finished the course with A and B marks was 63%.

BELIEFS: WHAT DOES RESEARCH SAY?

Broadly, mental states belong to three domains: *cognitive*, *affective* and *conative* (Hilgard 1980).

The cognitive domain involves states which are based on conscious evaluation of facts and logical reasoning, thus involving the phenomena of memory, perception and processing of information.

The affective domain gathers emotions, feelings, beliefs, and other related states, which are not based on purely logical foundations.

The conative domain is the most vague of the three — it involves mental states having high emotional charge for effort making, motivation and striving.

The affective and conative domains had not been taken seriously by mathematics educators up until 1980s (McLeod 1989, Lester 2003).

Mathematics learning and teaching had been thought to be of a purely cognitive nature, and the lack of progress in classrooms was attributed mostly to the lack of cognitive abilities and laziness of students.

Today, based on compelling evidence, has been acknowledged that the affective and conative domains play a significant, if not defining, role in learning and teaching of mathematics.

"Research that has examined the impact of students' beliefs on behavior has revealed significant relationships between beliefs and how students engage in learning and academic achievement."

"…qualitatively oriented studies have observed that students' beliefs appear to influence their engagement in learning with respect to the amount of time they work on a problem, the strategies they use to solve a problem, and their justifications as to what constitutes a correct response."

"[quantitatively oriented] studies have also found a significant relationship between students' beliefs and the types of behaviors they engage in while learning, and how those behaviors relate to achievement."

"Taken together, both approaches provide convincing evidence of the relationship beliefs have with learning and performance."

Muis, 2004

McLeod defined beliefs as the most stable states which have a significant cognitive foundation. That is, for a mental state to be defined as a belief, it should be held by an individual for a considerable time and, moreover, the individual should be able to articulate the logical reasons (which may be erroneous, of course) of why this mental state is present.

Beliefs have thus been shaped by the application of cognitive processes. In some cases these cognitive processes are biased and emotionally charged, generating potentially erroneous beliefs (see, for example Fiedler & Bless 2000).

In fact, when beliefs withstand the objective logical and empirical scrutiny and are found to be free from known biases, we can call such mental states as instances of *knowledge*.

One of the most practical ways to look at beliefs and their formation is through the lens of the theory of heuristics and biases, or, how this theory is also called, *behavioral economics* (Tversky & Kahneman, 1974).

From this point of view, erroneous beliefs are stable cognitive illusions akin to optical illusions.
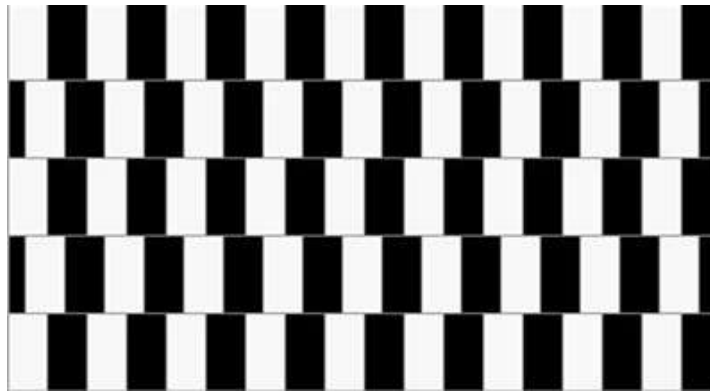
We believe something and we think that what we believe is true. Nevertheless, a factual examination can convince us that our belief is an erroneous one.

For example, an individual may have observed that he or she could not solve mathematical problems on several occasions. Obviously, several observations are not enough to make a conclusion about the general state of affairs, but because of the *availability heuristics* (Tversky and Kahneman, 1974), an individual makes a general conclusion that he or she is not capable to solve mathematical problems. Later this conclusion is bolstered by the *confirmation bias* (Nickerson 1998).

BELIEFS: WHAT DOES RESEARCH SAY?

But, beliefs are formed not only due to cognitive biases. Emotions are also involved:

"The arousal of emotions is of great relevance to belief-formation. […] There is evidence that belief formation under the influence of an emotion […] is removed from systematic reasoning."

"More importantly, and this is the heart of this point, cognitive shifts that are produced by emotions, occur in the absence of changes in relevant information: the subject "puts together" the evidence in a different way when her mood shifts."

"In considering someone else's self-deception calmly, we fail to understand how a person may come to hold the irrational belief, given the undermining evidence at her disposal."

(Lazar, 1999)

In short, erroneous beliefs are cognitive biases which are emotionally charged: they are "*beneficial*" to the belief-holder.

Host of related issues arise based on the beliefs being beneficial, for example *self-handicapping strategies, psychological games*.

CAN WE CHANGE NON-AVAILING BELIEFS?

"In summary, most studies examining whether students' beliefs can change as a result of specific changes in classroom instruction have found positive results. "

"…it appears that students' beliefs can change from non-availing to availing within a relatively short time"

"An important catalyst for change is an individual's awareness of his or her beliefs. An explicit awareness of beliefs may be a necessary factor for the perseverance of more availing beliefs regardless of the instructional environment. Without explicit knowledge of their beliefs, students may unknowingly revert to more non-availing beliefs."

"Teaching that focuses on speed, accuracy, and memorization of rules and procedures presented by the teacher and practiced in isolation is associated with beliefs that learning is quick, there is only one right answer, success requires innate ability, mathematical knowledge is unchanging and consists of isolated pieces of information, and the teacher is the source by which to justify mathematical knowledge."

"In contrast, constructivist-oriented approaches to teaching focus on establishing mathematics in meaningful and authentic contexts, engage students in collaboration and group activity to construct mathematical knowledge, are process oriented, and provide time for students to learn."

(Muis, 2004)

Based on the current research, there is hope that we can change non-availing beliefs, if we:

1. Foster constructivist-based instruction;

2. Avoid activities which would help students to corroborate their non-availing beliefs;

3. Speak to students about their personal epistemology;

4. Teach students to focus on effort and understanding, not on the grade.

5. Instructor must assess his or her own belief system.

What challenges do we have to implement these recommendations in our classrooms?

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