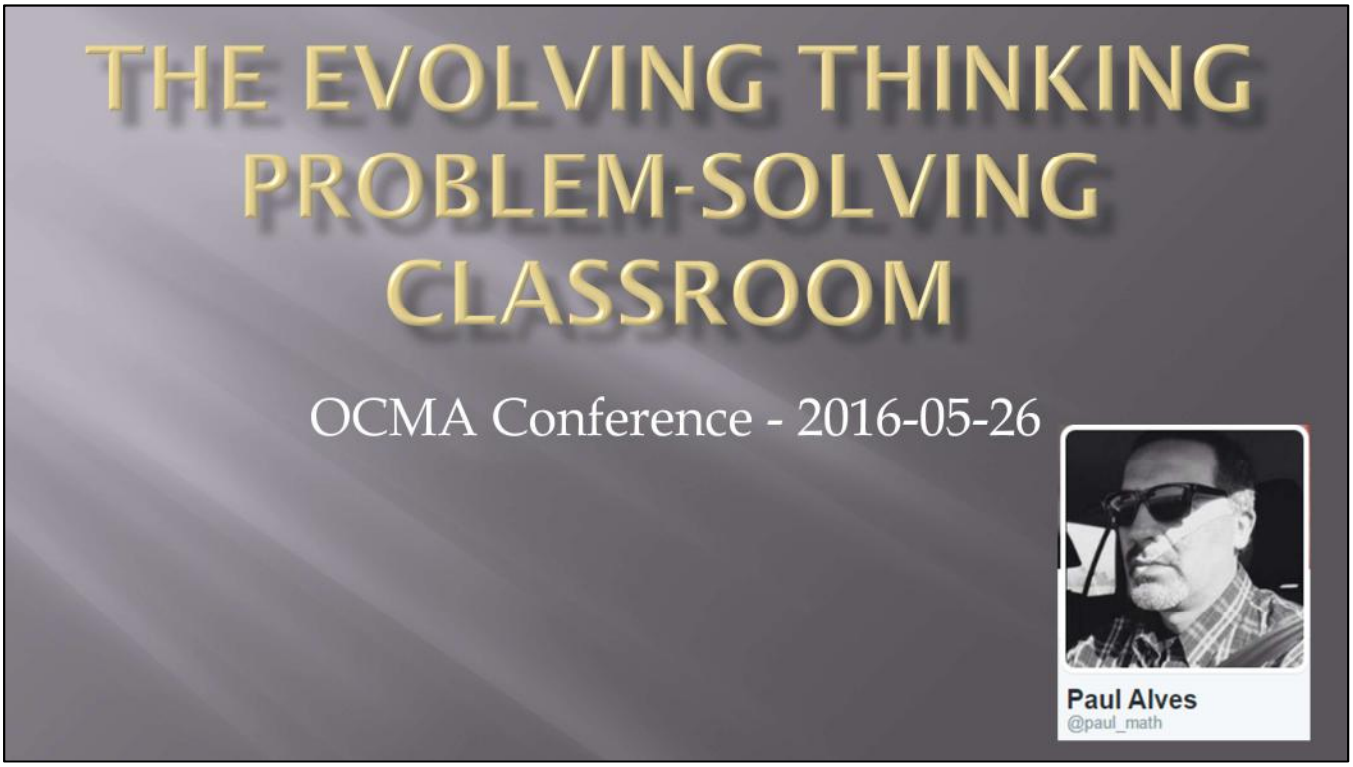
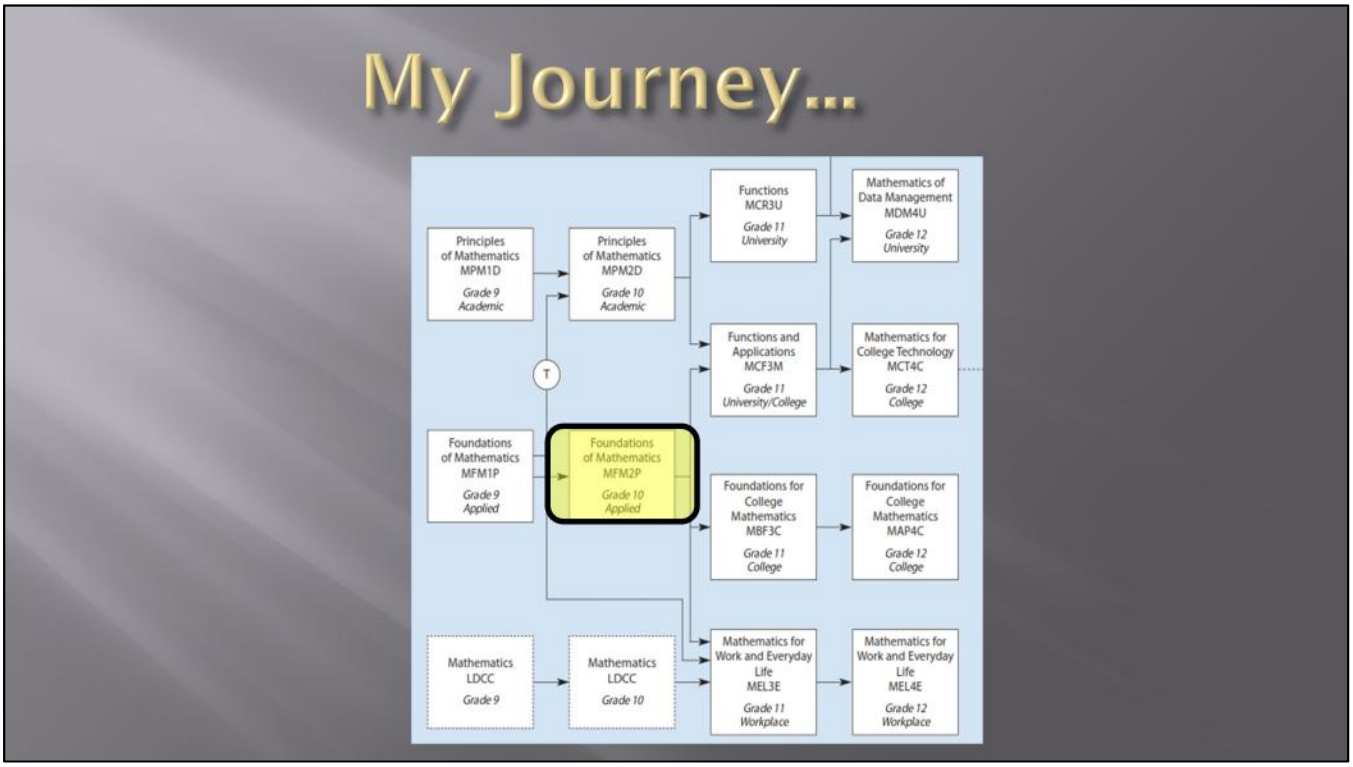


Department Head of Math at Fletcher's Meadow SS (Peel DSB).
Follow me on Twitter at @paul_math



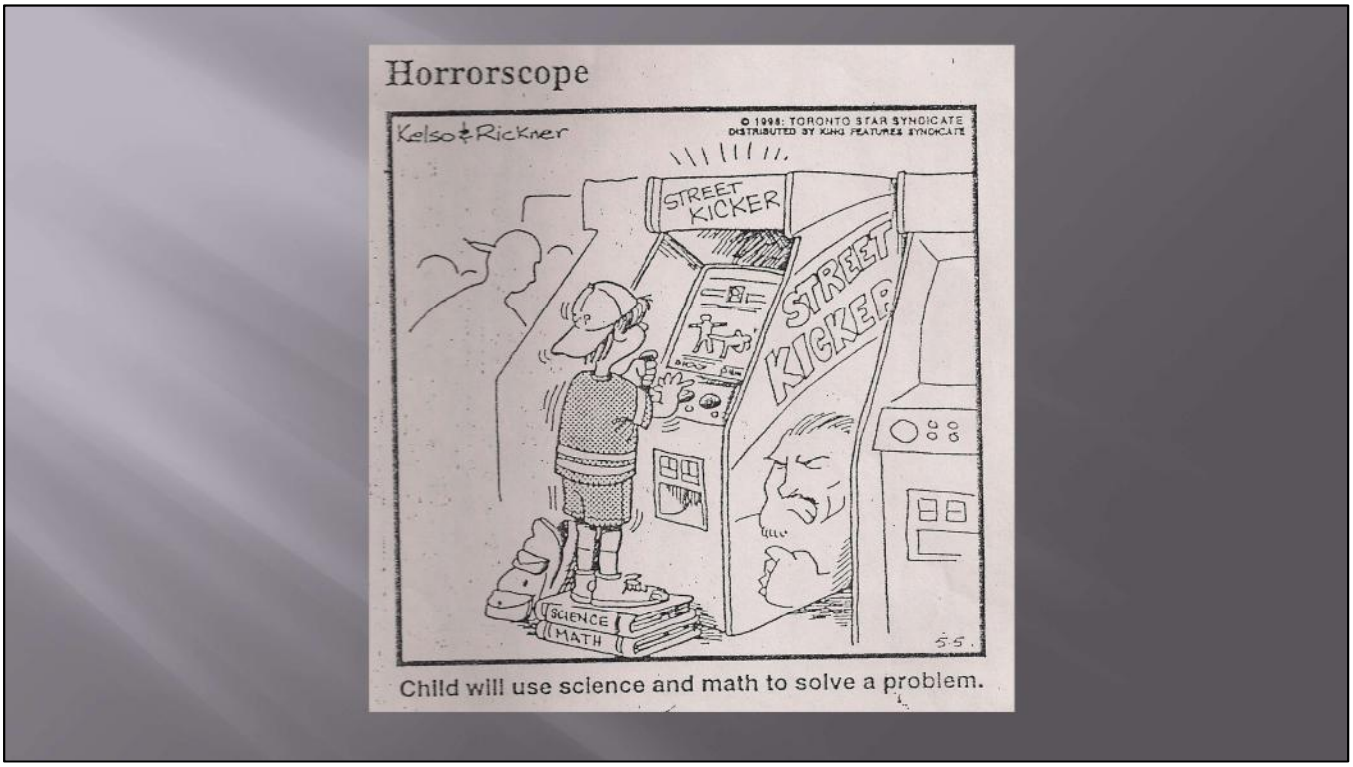
My own journey starts about 10 years ago in Gr. 10 Applied. A unique course in the high school program. You want it to be accessible enough to one group of students but rigorous enough for another group of students. Unlike some of the other Academic courses, there is a real differentiation of learners in this course.

Grade 10 Applied

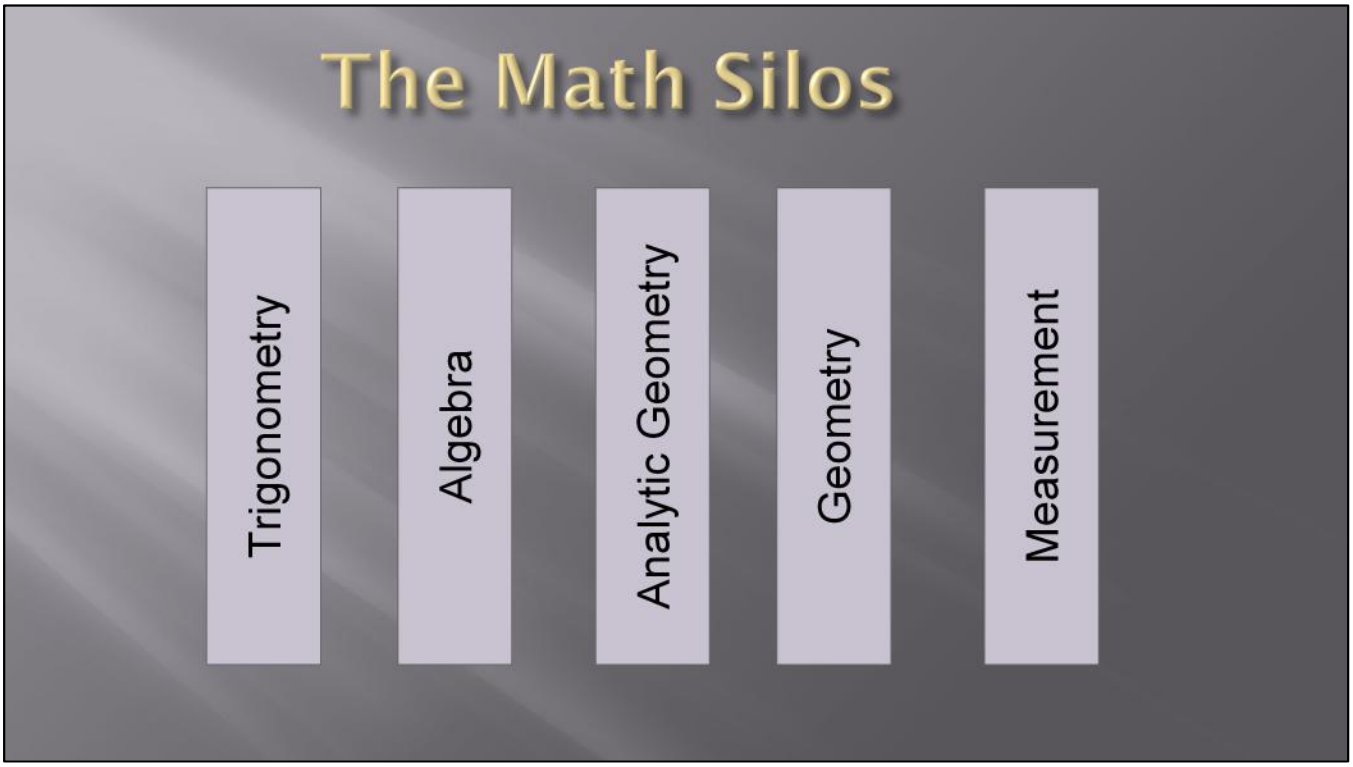
Grade 10 Applied: A challenging course since it appeals to three groups of students:

1. Group 1 – Wants to get 2nd of 3 of math credits
2. Group 2 – Wants to move to Grade 11 College with a possible College destination
3. Group 3 – Wants to move to Grade 11 University/College math (Mixed Math) with a possible University destination.

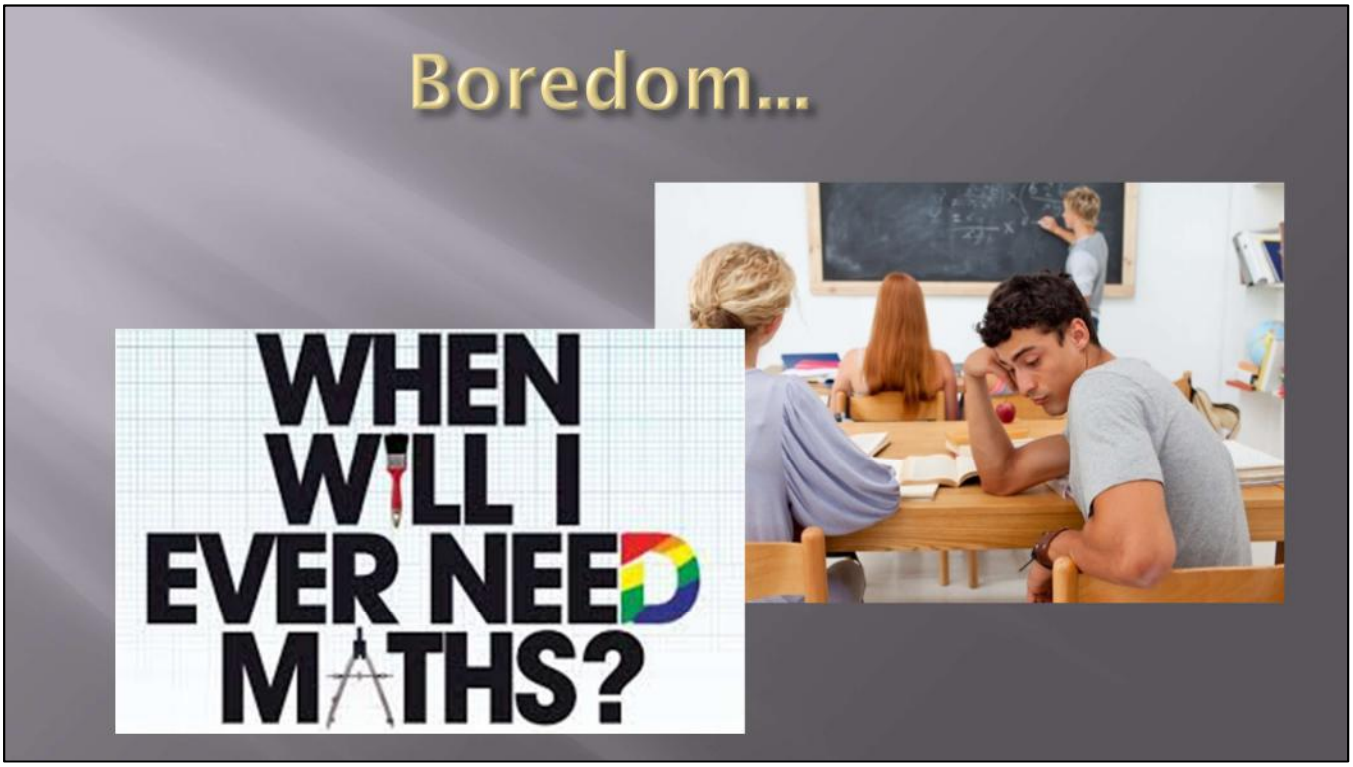
What did it look like when I was teaching for the first time? Very much like my own classroom experience. Students came in and I asked if they had any questions from homework. We took them up and usually I asked someone to post those solutions. I then taught my lesson which usually consisted of a note with guided examples. They practiced, I allowed some individual time, I then asked for a volunteer to share their solution. I would usually share an application of the concept being taught. Any time left was given over to getting started on the homework related to the day's lesson. There may have been a quiz...maybe.



This is how kids perceived what they were learning. There was very little that they could relate to. They didn't see the purpose to what they were learning. The problems were contrived to fit a very narrow application of the content.

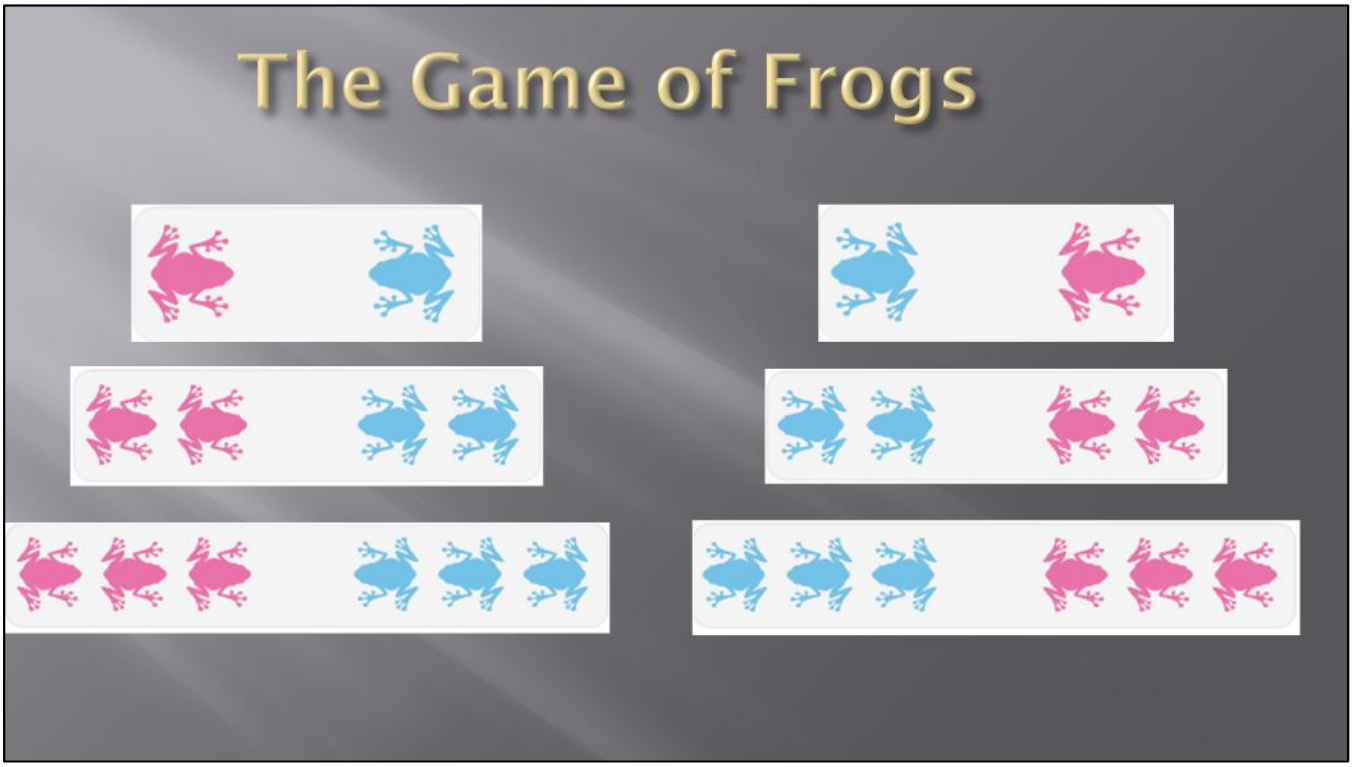


The students didn't see any connections between the content we were learning. We covered one unit and then moved on and so did that unit. They rarely retained the content. The course (in fact all the math courses) were modules of learning that rarely depended on the previous module. The curriculum from grade to grade would spiral (e.g. covering trig in Gr. 10 and going in more depth in Gr. 11) but the individual courses didn't. A common refrain from students was I really did well in unit xx but I hated unit xx.



The result in the classroom experience was invariably boredom. What I didn't anticipate was that I was the one who was bored! I clearly recall telling my course partner that I was bored teaching the course and if I was bored I couldn't imagine how the kids felt. This along with the results we were seeing was the impetus to change.

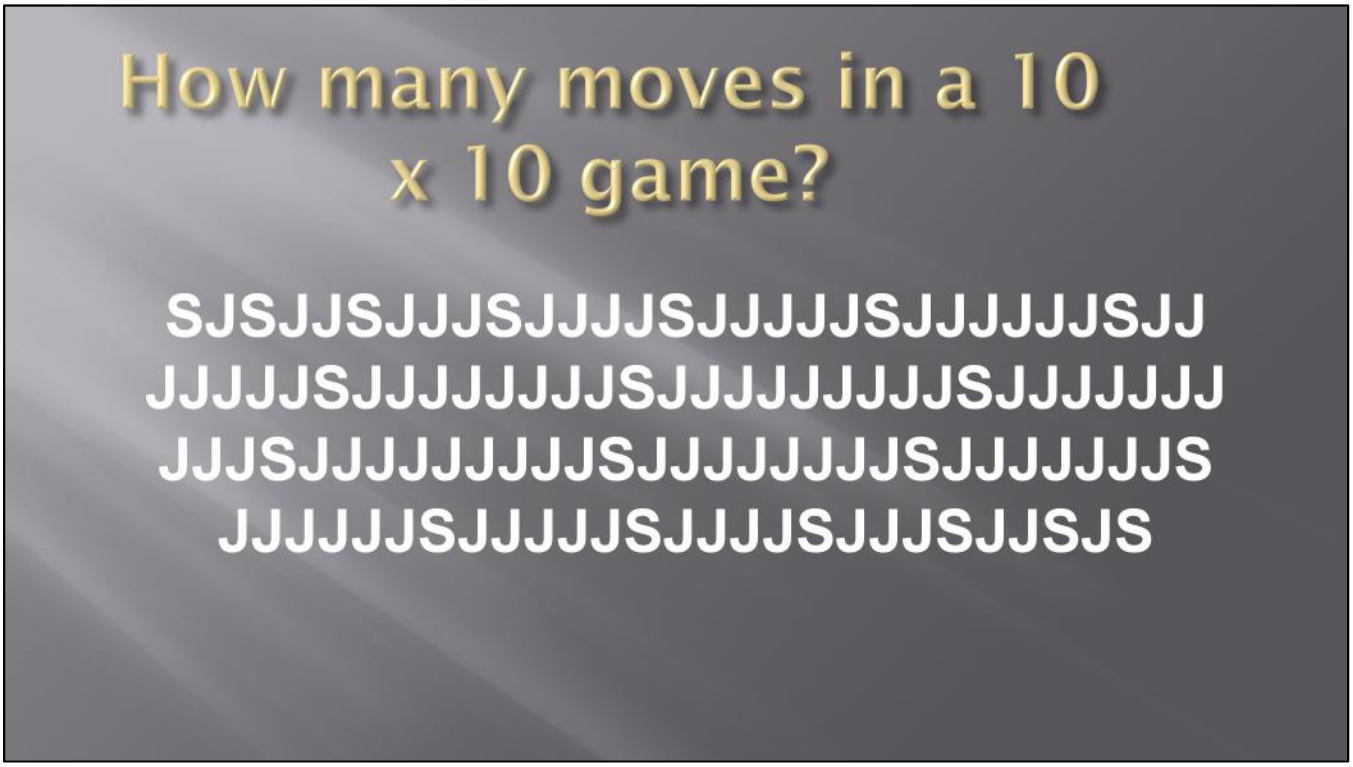
We decided to move to a problem based approach to the concepts. We did our best to find problems that that connected to the content that would engage students and initiate their prior learning and intuitive understanding.
Ex. The fruit problem provided a visual way to start the linear system unit
Ex. The tug of war problem connected to algebra
Ex. Tetrahedral fractal connected to similarity and exponents.
Ex. My personal favourite...the Game of Frogs.



The rules are simple. Start with one frog on each end with a space in between. Your goal is to make them switch spots (start position is on the left and end position is on the right) but you are constrained by these rules.

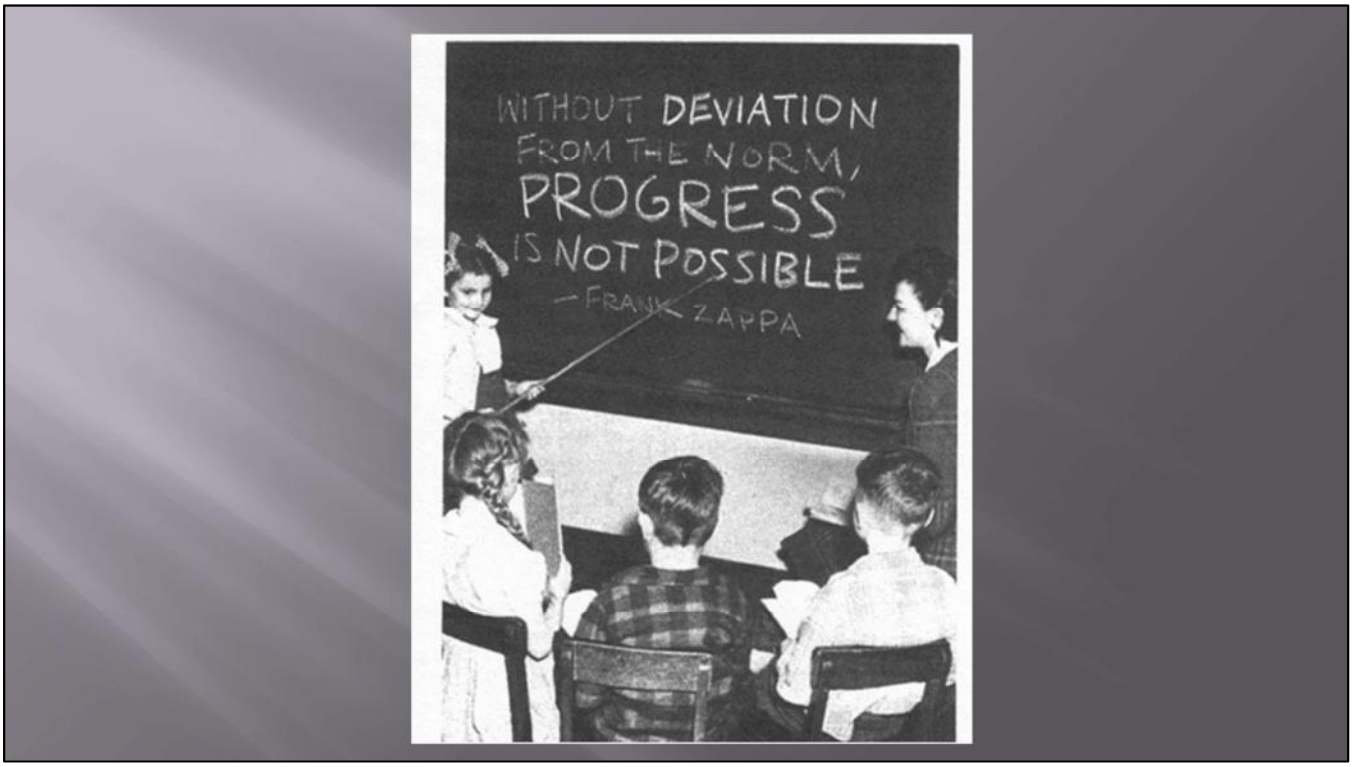
1. You can slide to an empty space adjacent to the frog.
2. You can jump ONE frog if the space on the other side is empty.
3. NO MOVING BACKWARDS.

How many moves will it take for the 1 by 1 game? 2 by 2 game? 3 by 3 game? 10 by 10 game?



I knew the students were engaged when one of my more disengaged learners presented this solution to the 10 by 10 game. He had seen a pattern in the sequence of jumps and slides that helped him determined the total number of moves.
For an interactive version of the game: https://nrich.maths.org/1246

The impact it had on the classroom environment was significant. The classroom became a louder place. But not louder because students were off task and distracted. It became louder as students worked TOGETHER to solve problems and tried justifying strategies and solutions to their peers, to me, to the classroom. A collaborative approach to solving authentic and rich problems was valued over individual struggle on contrived problems.



Using problems to engage students, to provoke their thinking and to bring out the math was something that didn't happen every day but it happened much more often than the traditional math lesson that we had delivered in the past.

Another example...using a context that can engage students. Right graphic: In 2008 , Michael Fournier attempted to break the freefall record. We followed his attempt (scheduled to happen in Saskatchewan) and tried to convert his jump into a benchmark students could relate to – How many CN Towers would this be? On the date of the jump, the balloon detached from the capsule he was using to ascend and floated away. Left graphic is the successful jump of Felix Baumgartner in 2012: https://www.youtube.com/v/FHtvDA0W34I Students were able to use proportional reasoning and it introduced imperial units to them in Gr. 10 Applied curriculum.

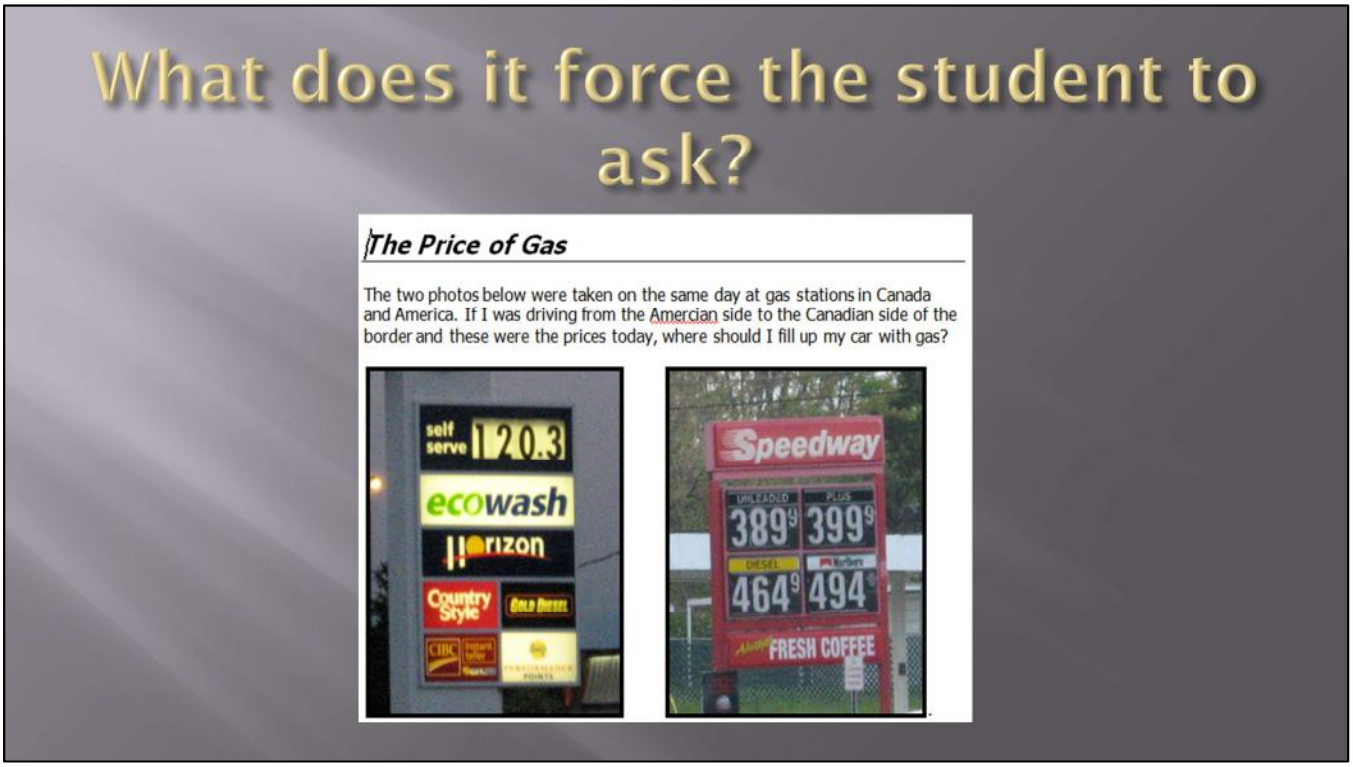
Using rich tasks

Mr. Alves's Gas Problem

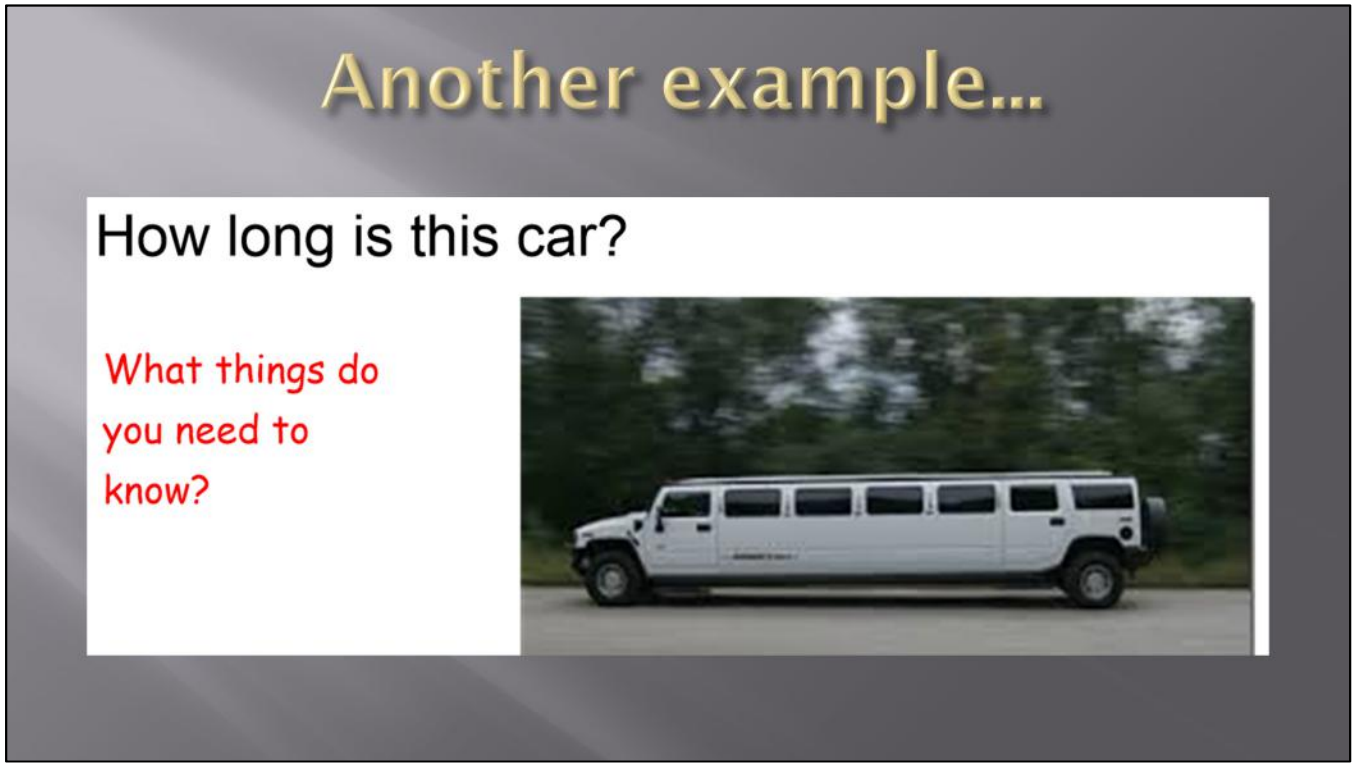
The Price of Gas

The two photos below were taken on the same day at gas stations in Canada and America. If I was driving from the Amercian side to the Canadian side of the border and these were the prices today, where should I fill up my car with gas?

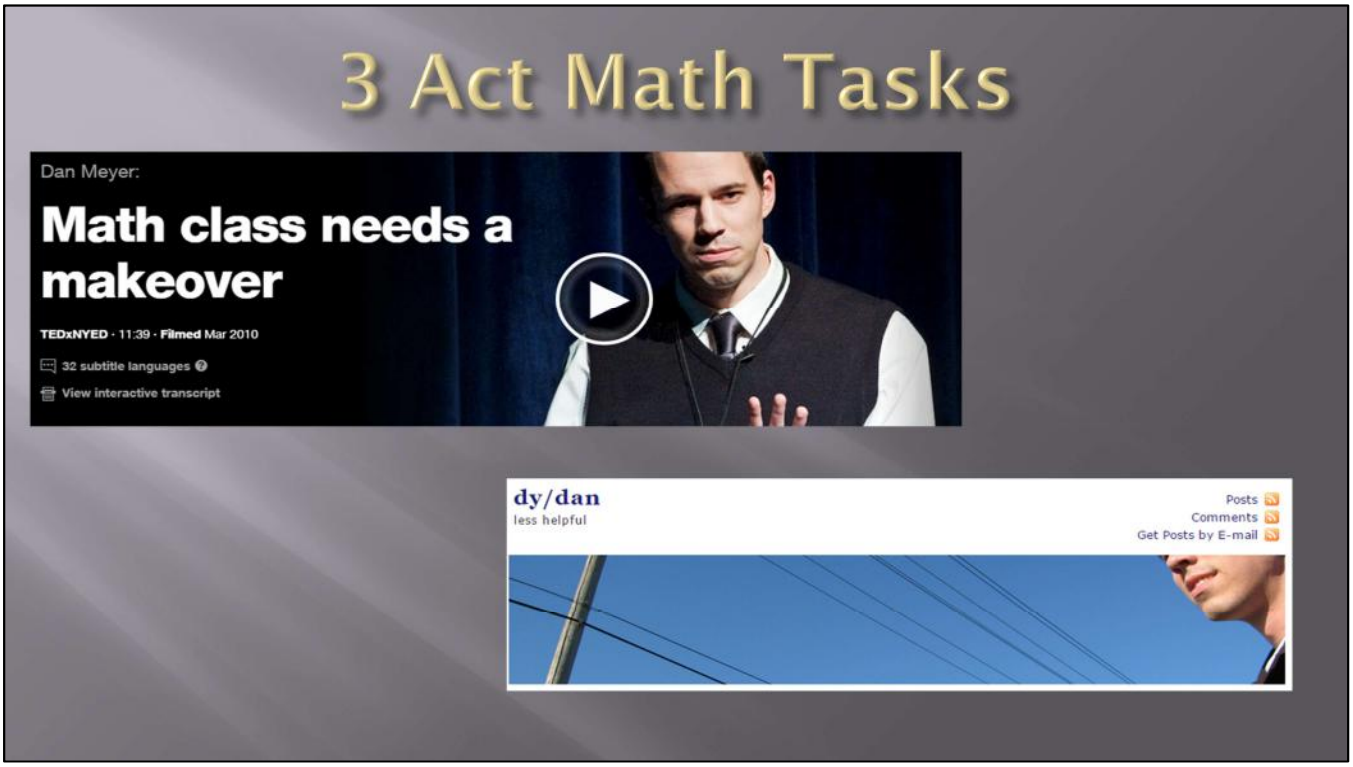
Another example...An unfortunate name for this task but students were required to determine whether I should have purchased gas on the US or Canadian side of the border. Required students to consider what the variables are that they needed to consider. Students used proportional reasoning and their understanding of imperial and metric systems to solve the task.



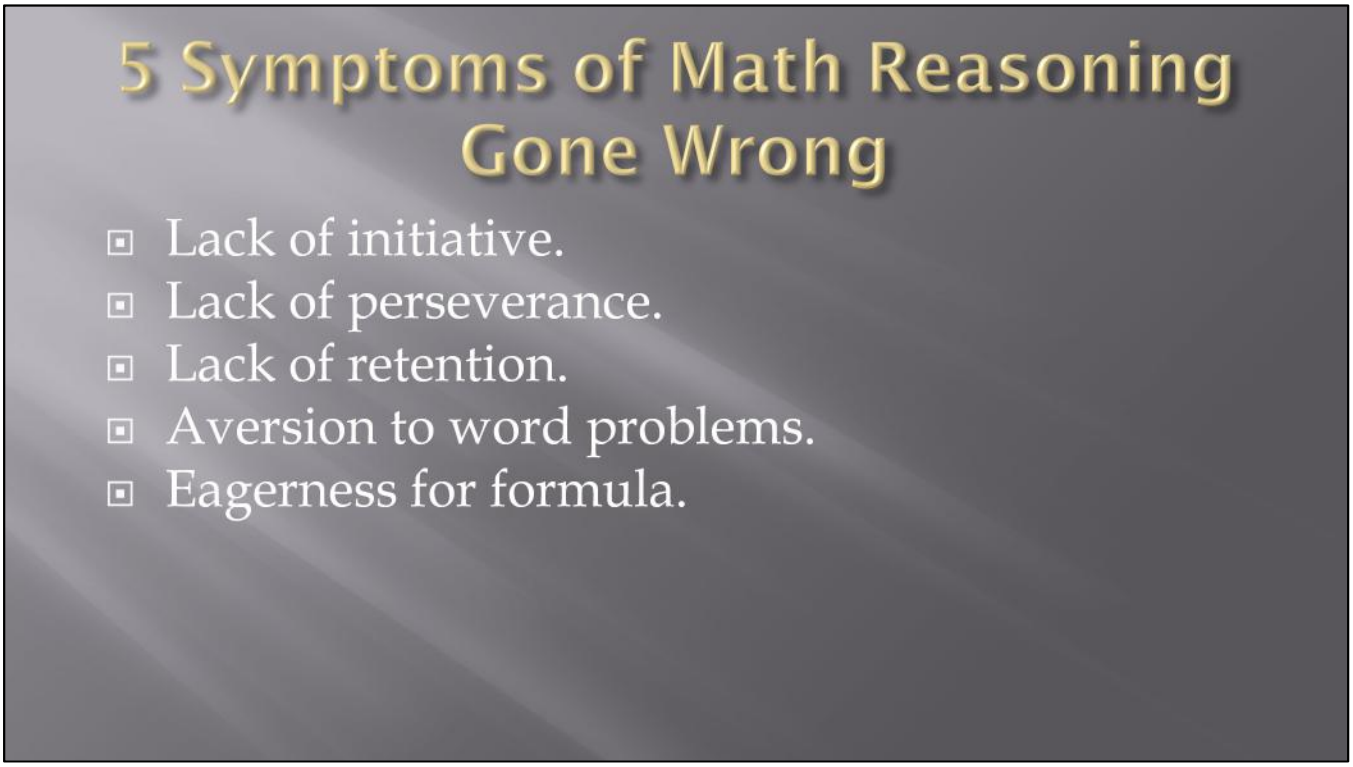
What do I need to know to solve this problem? Again, students needed to consider what are necessary pieces of information (and what is unnecessary). For example, how big is the gas tank in the car; what are the units for the prices; how do we read the prices; how are gallons and litres related; what is the exchange rate; etc.



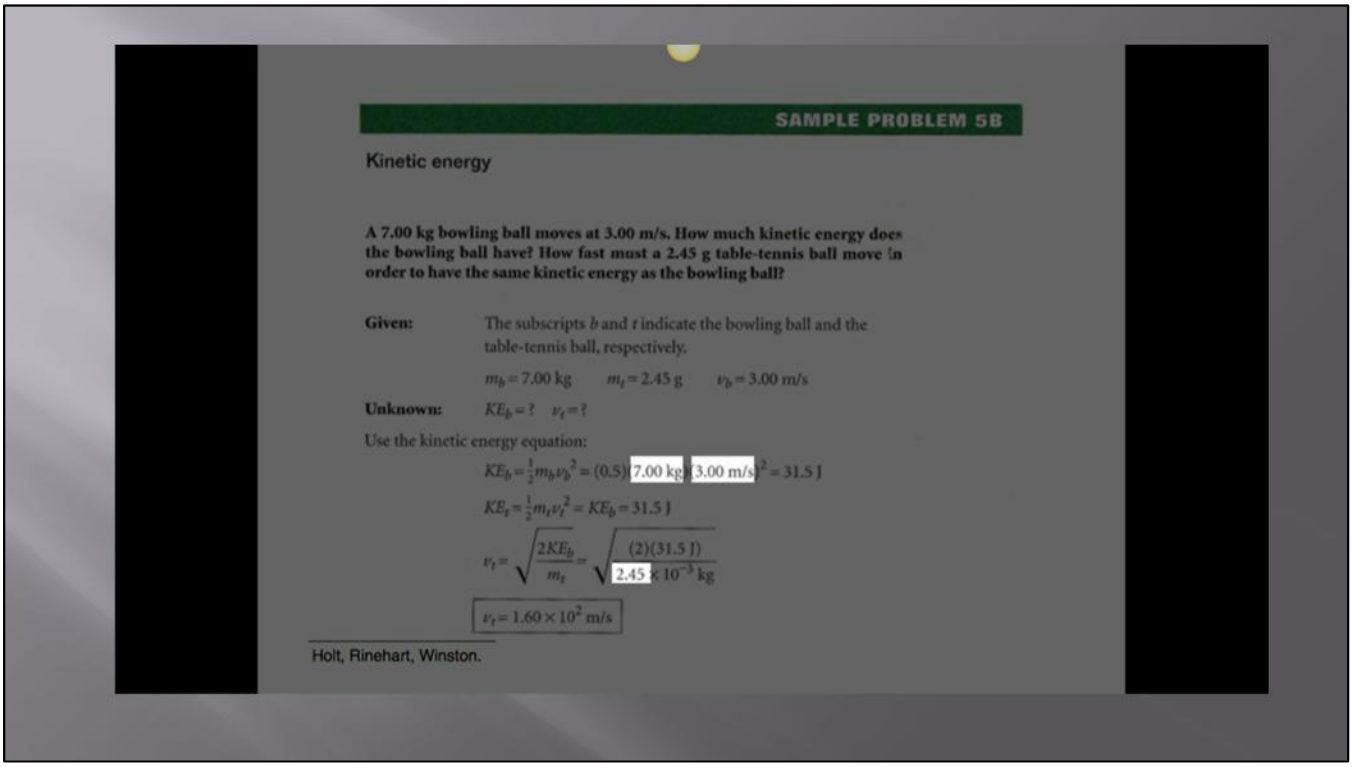
Last example...as an example of similarity and incorporating measurement again, I project this image and asked students to determine the length of this limo. First part was to think of a reasonable estimate of the length – is it as long as the classroom? Estimation and reflecting on the reasonable of an answer are things we need to value more in the math classroom.



Dan Meyer really crystallized the issue with the way we have been approaching math problems for some time in his 2010 TED Talk (https://www.ted.com/talks/dan_meyer_math_curriculum_makeover?language=en). He deconstructs a traditional problem and assesses the issues with the way we scaffold problem solving for students. He proposes a new approach – 3 Act Math Tasks.
You can follow Dan and look at his math tasks on his blog: http://blog.mrmeyer.com/



Dan identifies five symptoms that you're doing math reasoning wrong in your classroom.
Initiative – Students don't self start. They wait for you to re-explain what you just did in your lesson.
Perseverance – Students stop as soon they encounter an issue when working through a problem.
Retention – You have to re-explain lessons days later.
Word Problems – Students hate word problems. They avoid them because of issues with decoding and selecting the appropriate tool/strategy to solve.
Formulas – They want a process or formula to follow.



SAMPLE PROBLEM 5B

Kinetic energy

A 7.00 kg bowling ball moves at 3.00 m/s. How much kinetic energy does the bowling ball have? How fast must a 2.45 g table-tennis ball move in order to have the same kinetic energy as the bowling ball?

Given: The subscripts *b* and *t* indicate the bowling ball and the table-tennis ball, respectively.

$m_b = 7.00\ \text{kg} \qquad m_t = 2.45\ \text{g} \qquad v_b = 3.00\ \text{m/s}$

Unknown: $KE_b = ? \quad v_t = ?$

Use the kinetic energy equation:

$$KE_b = \tfrac{1}{2} m_b v_b^2 = (0.5)(7.00\ \text{kg})(3.00\ \text{m/s})^2 = 31.5\ \text{J}$$

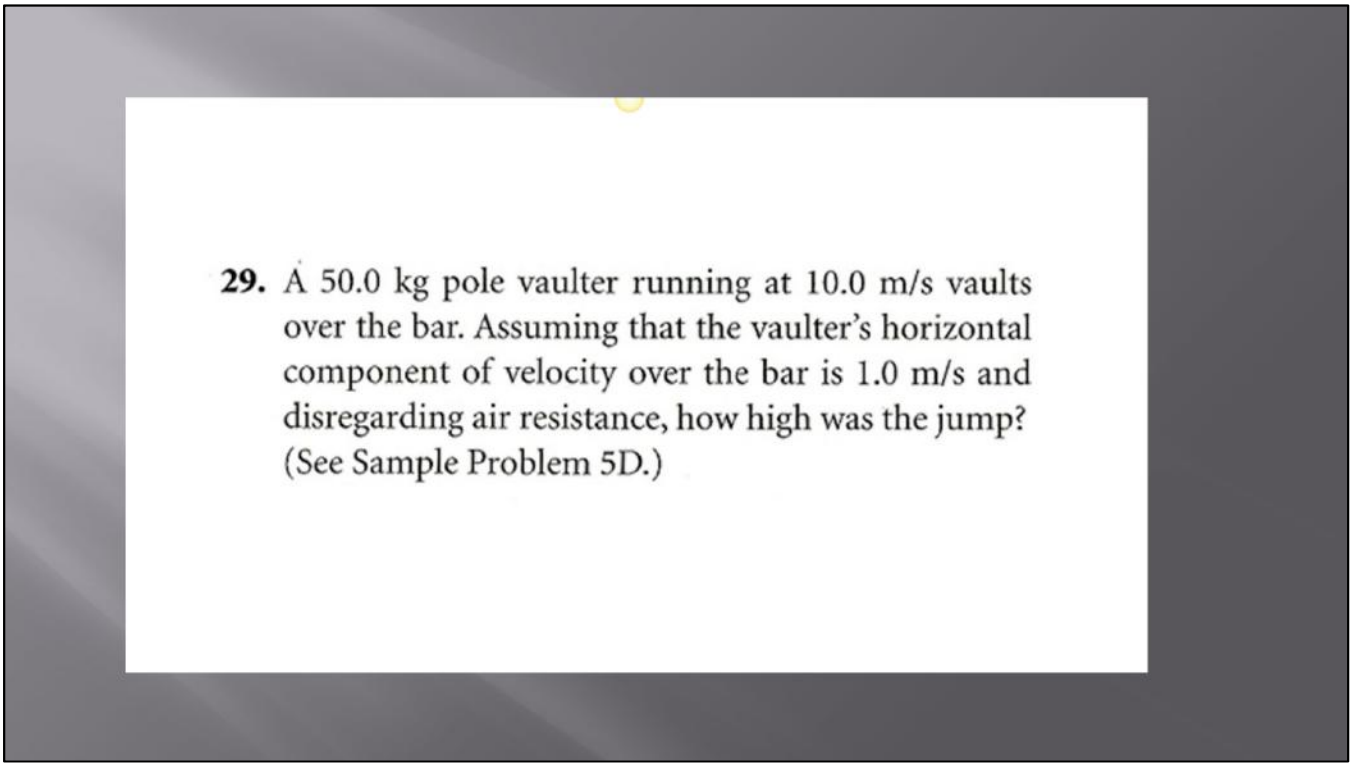
$$KE_t = \tfrac{1}{2} m_t v_t^2 = KE_b = 31.5\ \text{J}$$

$$v_t = \sqrt{\frac{2KE_b}{m_t}} = \sqrt{\frac{(2)(31.5\ \text{J})}{2.45 \times 10^{-3}\ \text{kg}}}$$

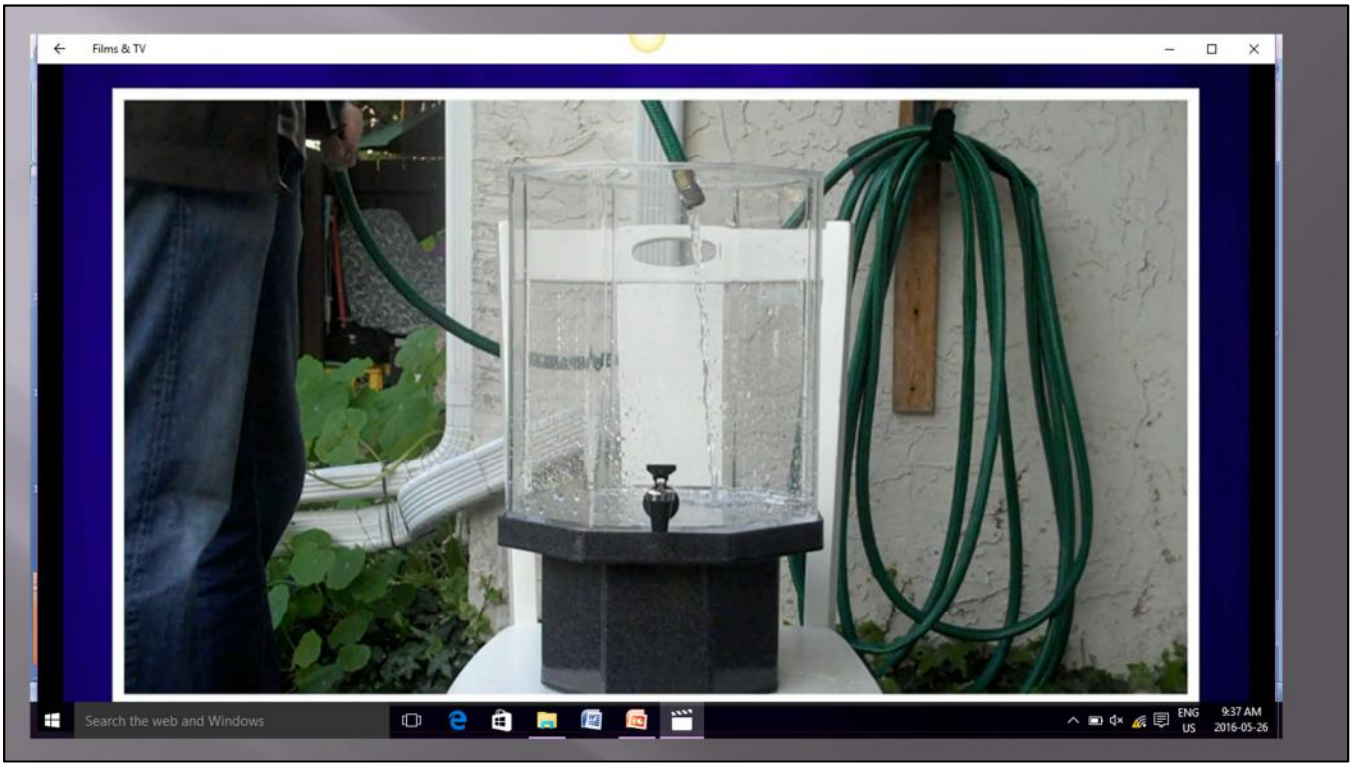
$$v_t = 1.60 \times 10^2\ \text{m/s}$$

Holt, Rinehart, Winston.

A sample from a Physics text from the TED Talk. The formula is provided. The unknowns and knowns are identified and then substituted nearly into the formula.

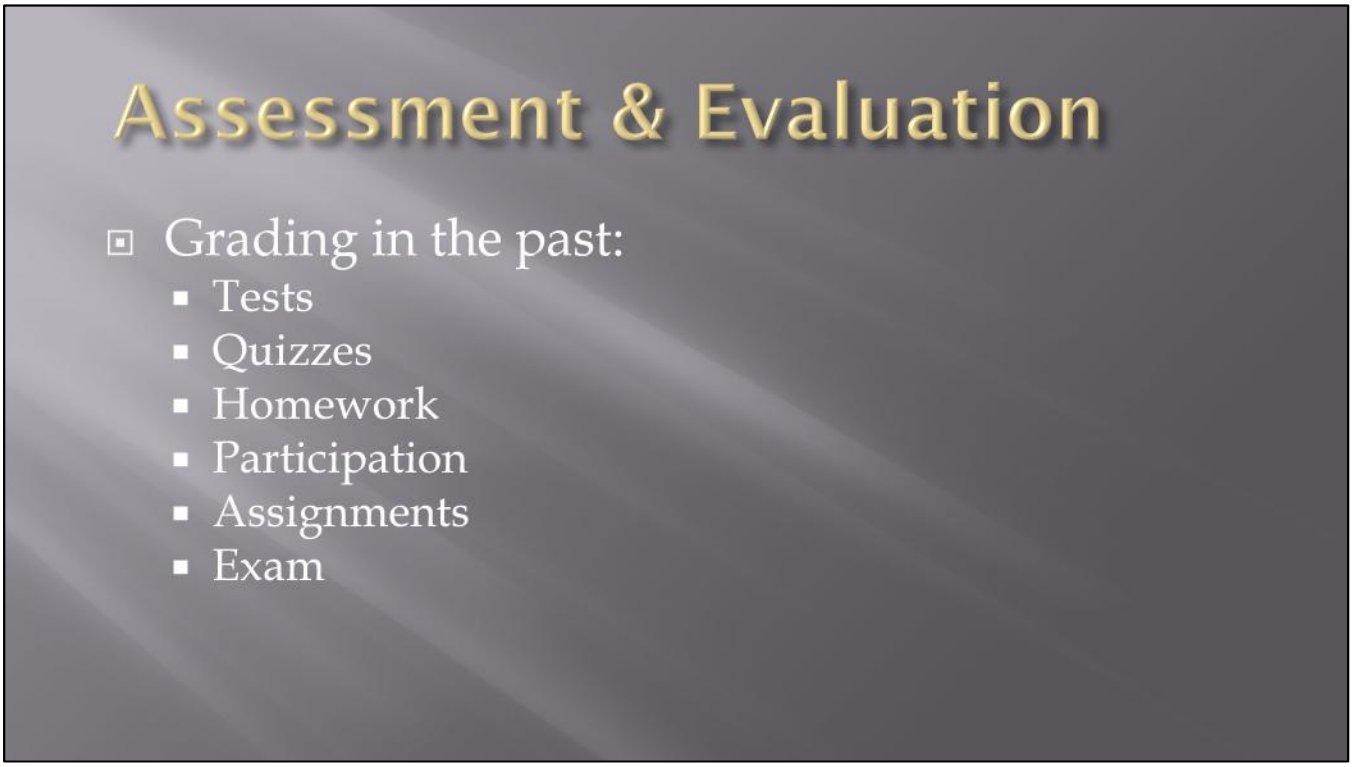


When students are assigned follow up practice, the problems mimic the examples and sometimes a hint is provided that alerts them to the example that is identical to the problem. Students come to depend on this model for solving problems. They want to pigeon hole the problems – this problem is like that one. Any problem that deviates from ones provided in class (or in the resource) creates an issue. The common refrain from a test – I didn't get that problem because you didn't teach us one like it. **They are inflexible in their thinking because we made them rigid in their learning!**



A screen shot from the TED Talk. The 3 parts to a 3 Act Math Task:
Act 1 The Teaser – "**Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."** This could be a video or picture or just a context. The key is to engage the learner's curiosity. What questions do they ask? What do they wonder?
Act 2 The Plot Thickens – "**The protagonist/student overcomes obstacles, looks for resources, and develops new tools."** Students need to consider what they need to know to determine the solution to their question. What tools/strategies will they need to solve the problem?
Act 3 Resolution – "**Resolve the conflict and set up a sequel/extension."** Students determine how well they did by seeing the resolution to the problem (e.g. the end of the video).
We do Act 2 a lot in our classrooms but we have been missing or not truly developing Act 1 and 3.

There are many Canadian educators developing 3 Act Math Tasks targeted to our expectations.
Kyle Pearce – https://tapintoteenminds.com/
Jon Orr - http://mrorr-isageek.com/
Mary Bourassa - http://marybourassa.blogspot.ca/
Amy Lin - https://amylin.me/
Alex Overwijk - http://slamdunkmath.blogspot.ca/



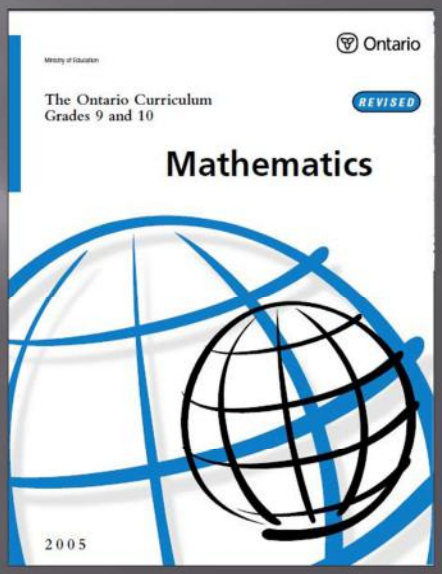
What impact does teaching through problem based learning have on A & E? A bit of a history lesson here. Pre-1999, students marks were determined according to a breakdown that used some or all of these tools of evaluation. There may have been inclusion of projects or something else here and there but for the most part, this is the way it looked.

Some context

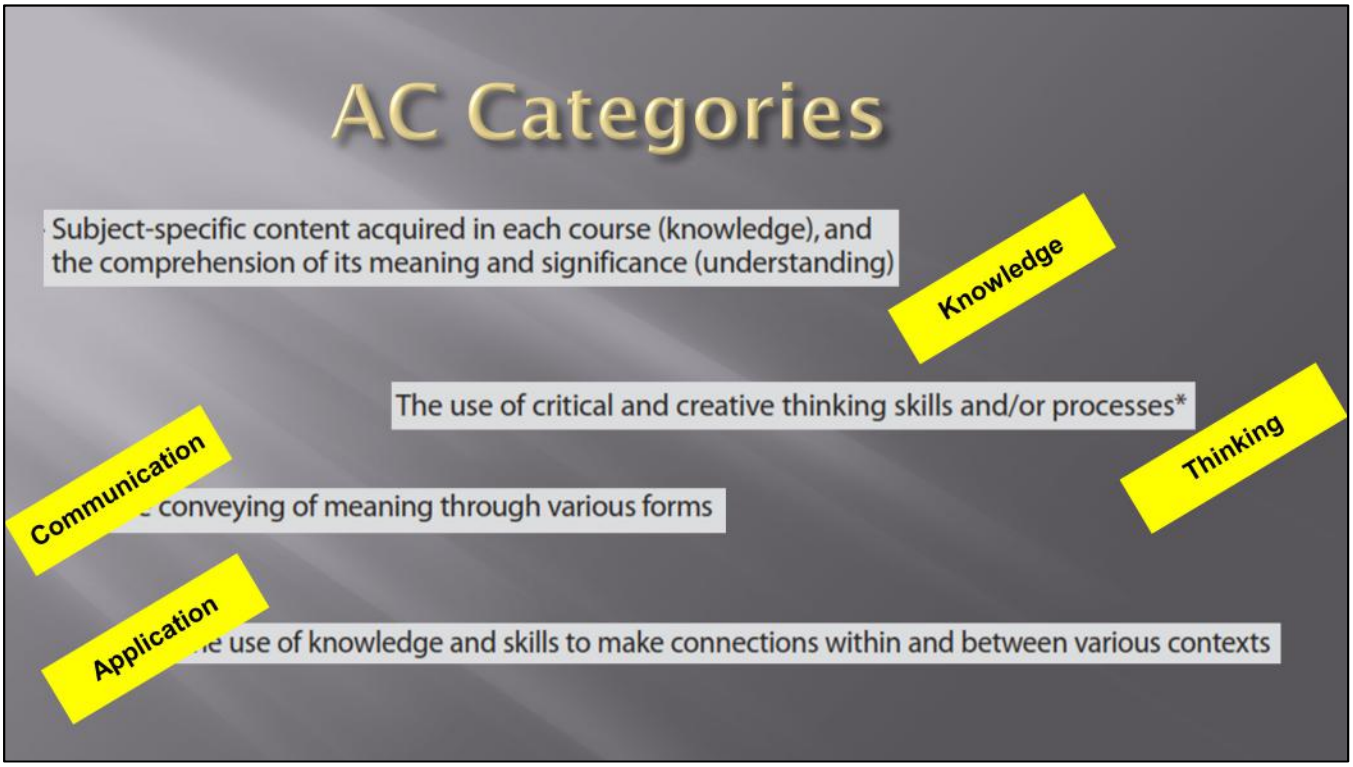
In Ontario, student marks are broken down into four categories:

- Knowledge and Understanding
- Application
- Communication
- Thinking

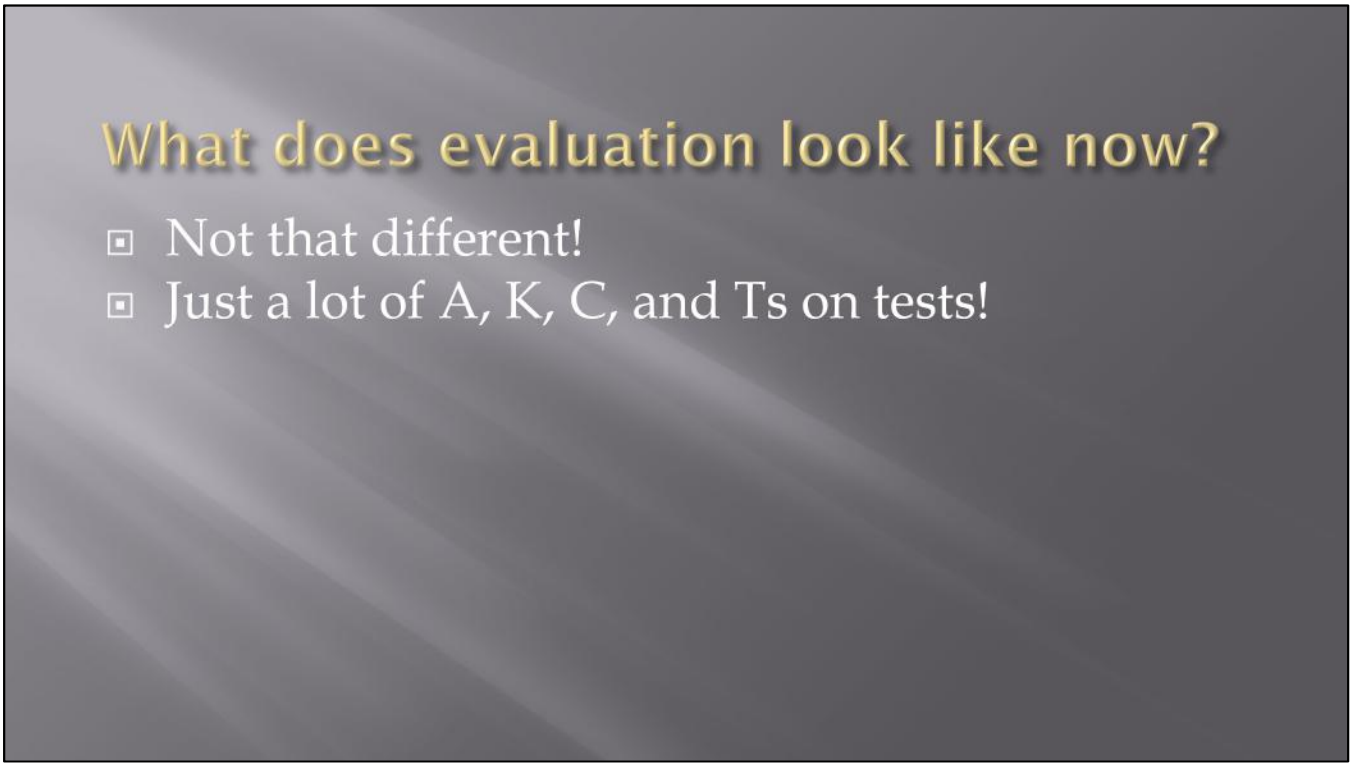
These are called the achievement chart categories.



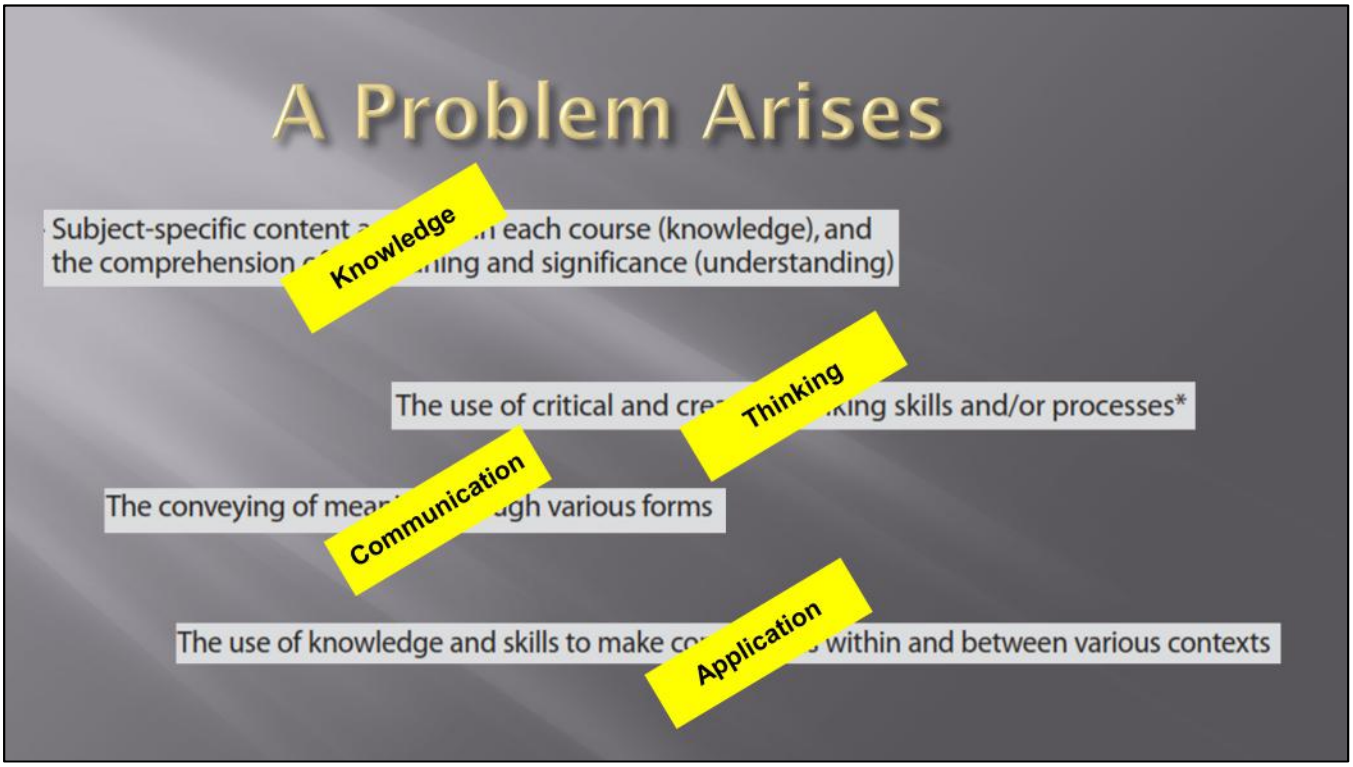
When the curriculum was revised in 1999, the manner in which students were evaluated was changed to reflect their achievement in these categories. Students marks were determined by 70% term work (some of these evaluation tools were similar to what was used prior to 1999) and 30% summative work (primarily pencil & paper exams).



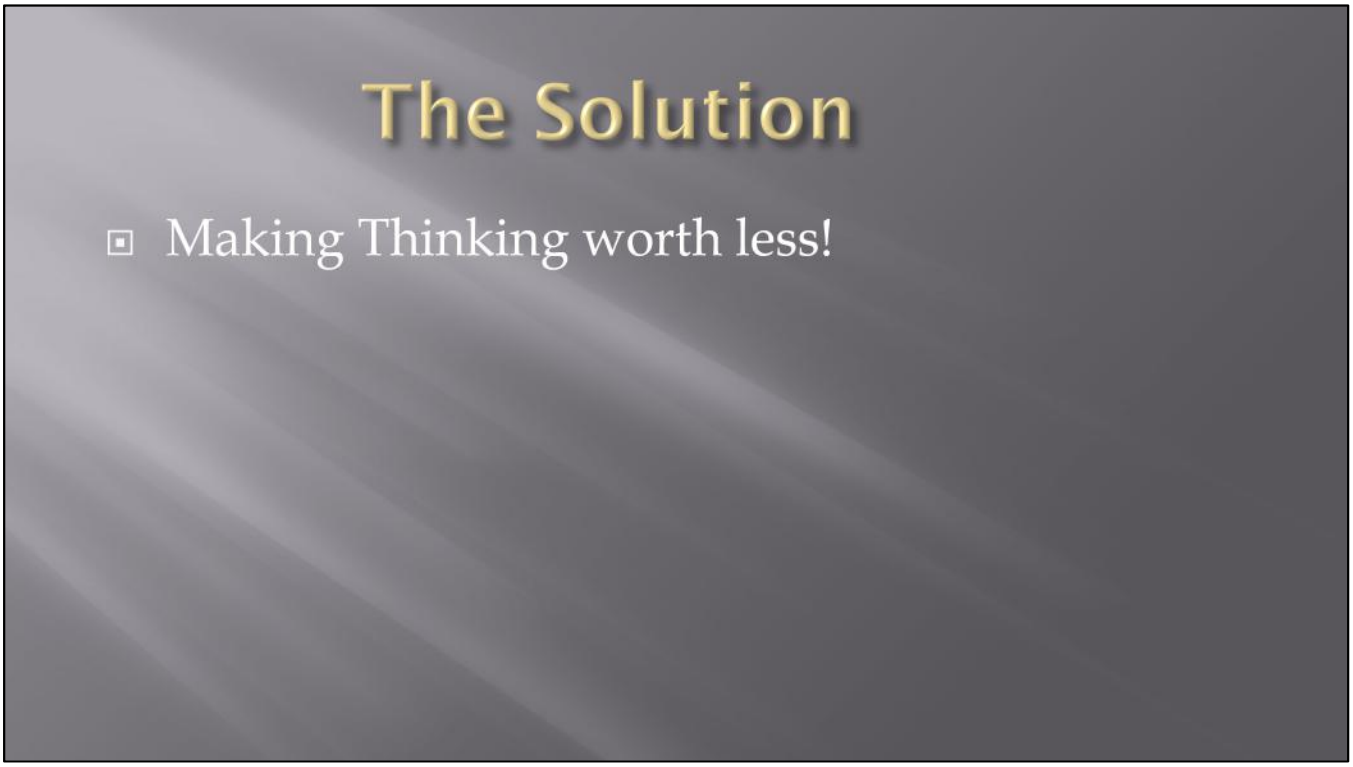
What are these categories?



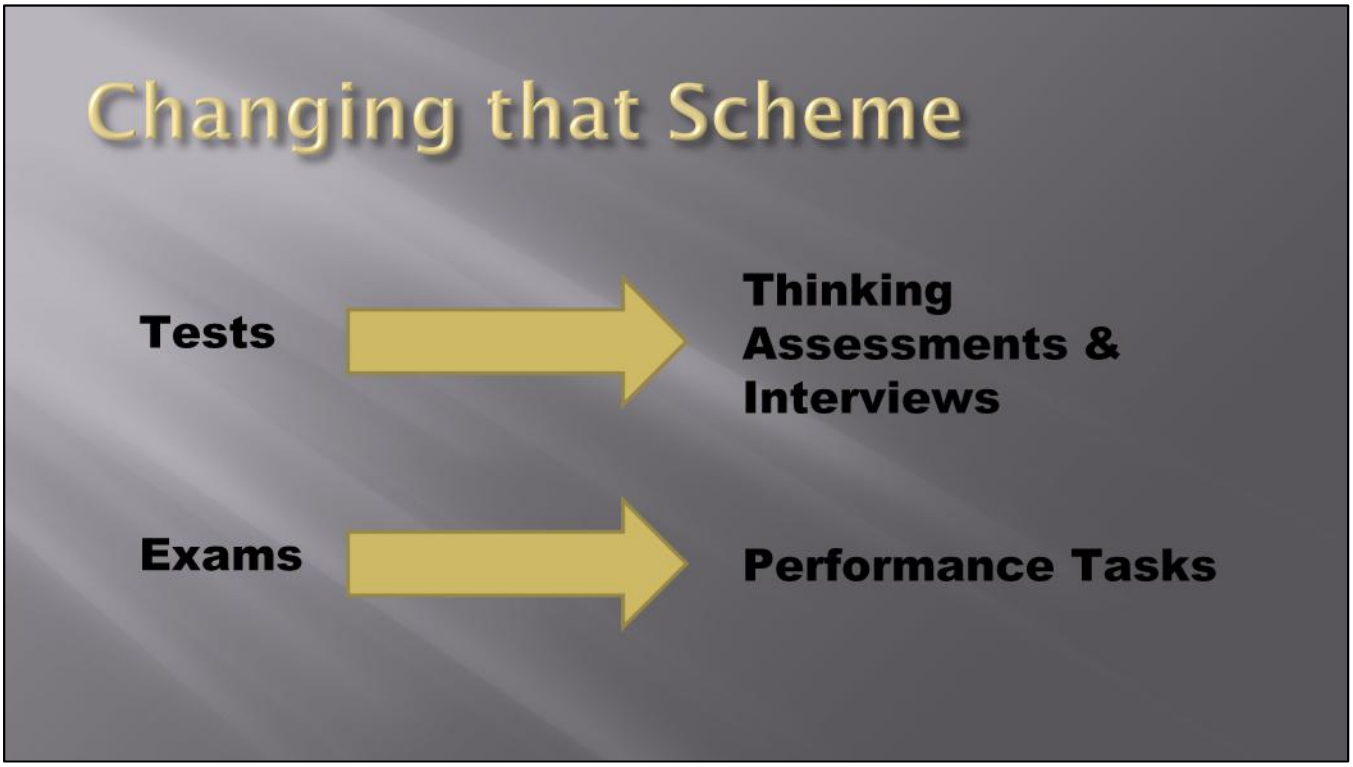
How did our tools for A & E change to reflect this new approach to evaluating students? Not a lot. For the most, many HS math teachers took what they used in the past and categorized the questions into the achievement chart categories.



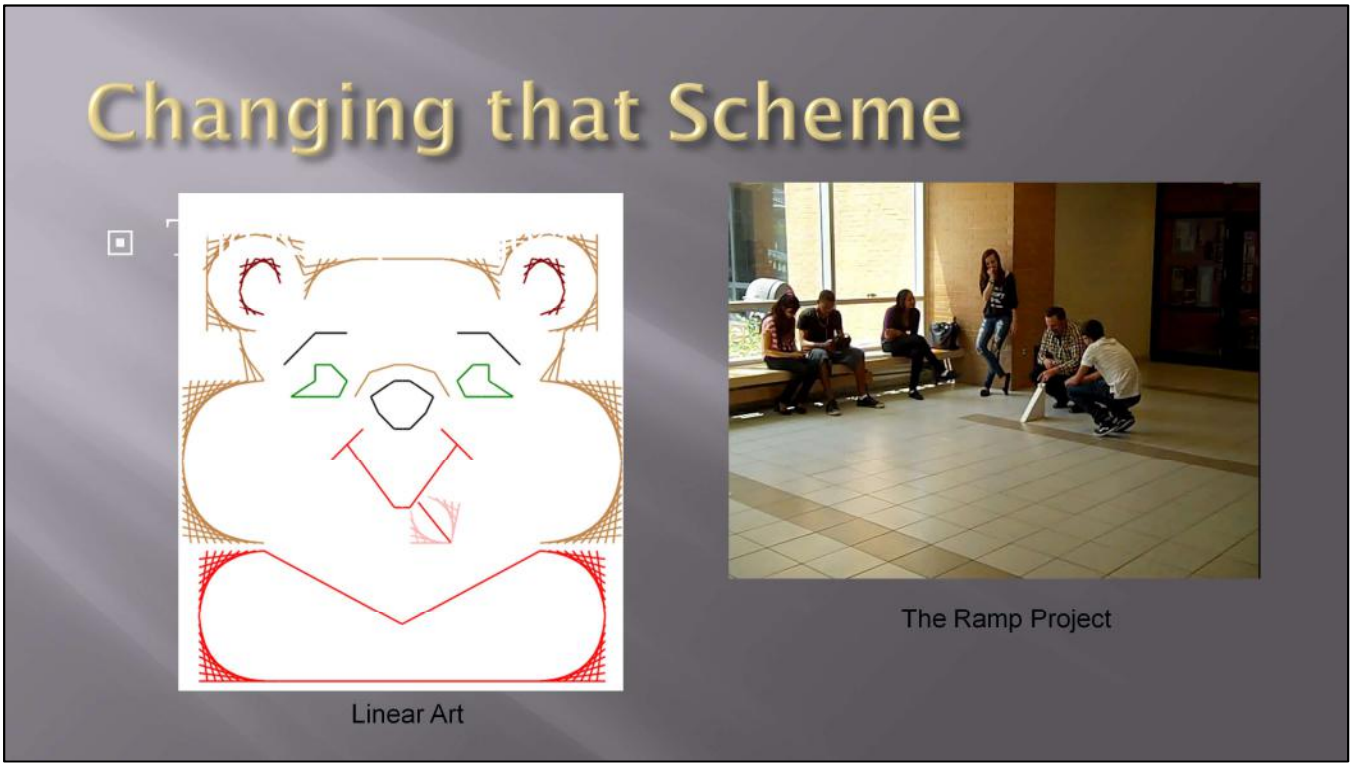
What did we see? Student struggles in problem solving emerged front and centre! In the past they were masked by the inclusion of marks that were awarded to participation, homework or just an overall test mark but now they came into glaring focus.



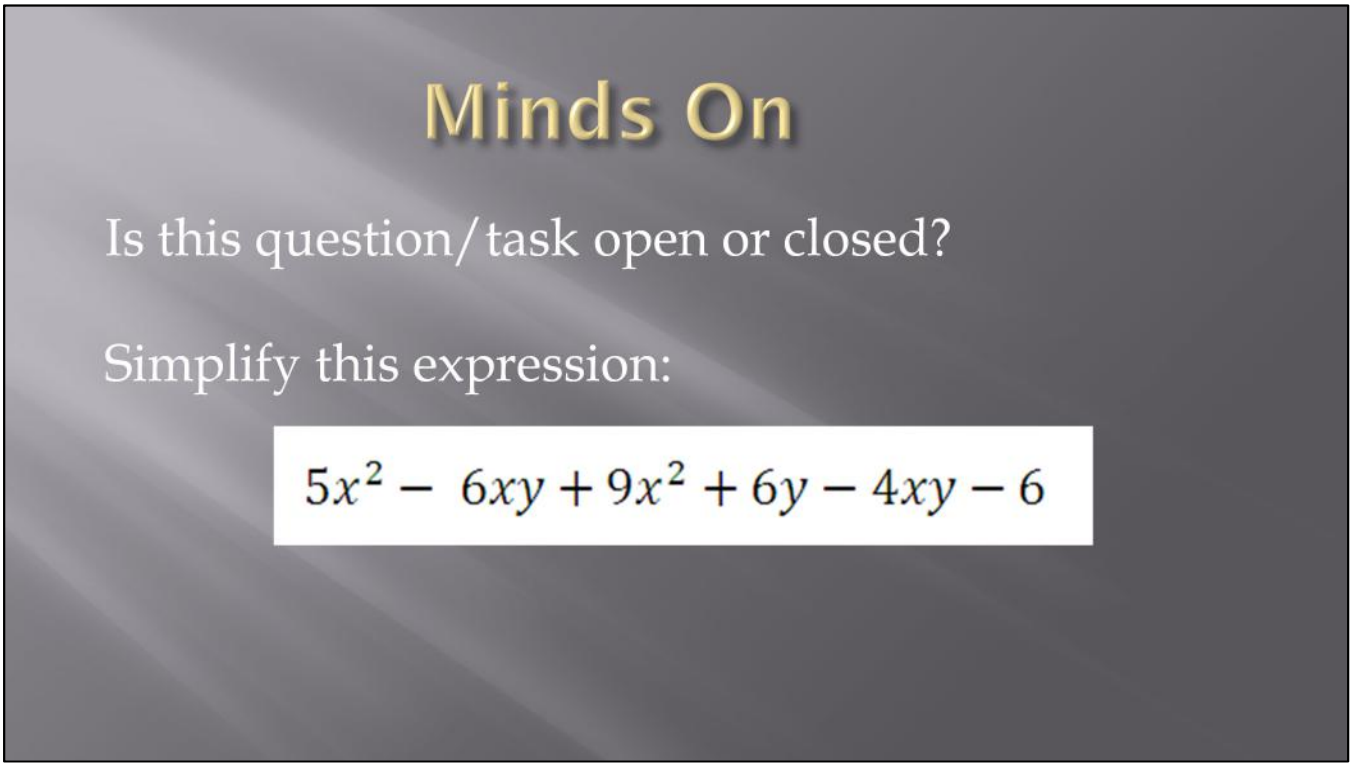
Our response? Make the problem solving category worth the least in the mark breakdown! An incredible irony that in math – a discipline that is all problem solving – we now value this category the least.



What tools do we use that truly reflect the change in the curriculum and honour teaching through problem solving?
Tests and exams are still there but they may not be the best tool for authentic PS. Thinking assessments that focus on an authentic open-routed or open-ended problem can be used. Interviews are an alternative to the traditional paper & pencil test that allow students to communicate their understanding of solving a problem. Performance Tasks that require students to put together their understanding of multiple concepts to achieve a specific goal.



Examples for Gr. 10
Linear Art Project – Students have to create an original design using only lines and provide the equations to a selection of the lines. An interview is an option to require students to justify the use of a selection of lines.
The Ramp Project – A performance task that incorporated multiple strands.



A strategy for allowing all students to demonstrate their thinking is the use of open problems in instruction and assessment. This is better than using the “C-level” problems in the texts which is what we had been doing. Students needed to see the “in” for these challenging problems or else they were stuck outside! Many times students would just hand in blanks for these problems. We needed to look at the problems and ask were they open or closed? This is a closed question.

Minds On

Is this question/task open or closed?

The perimeter of a rectangle is $10x^2 + 16x - 12$. Create a rectangle that would have this perimeter if each side is a trinomial.

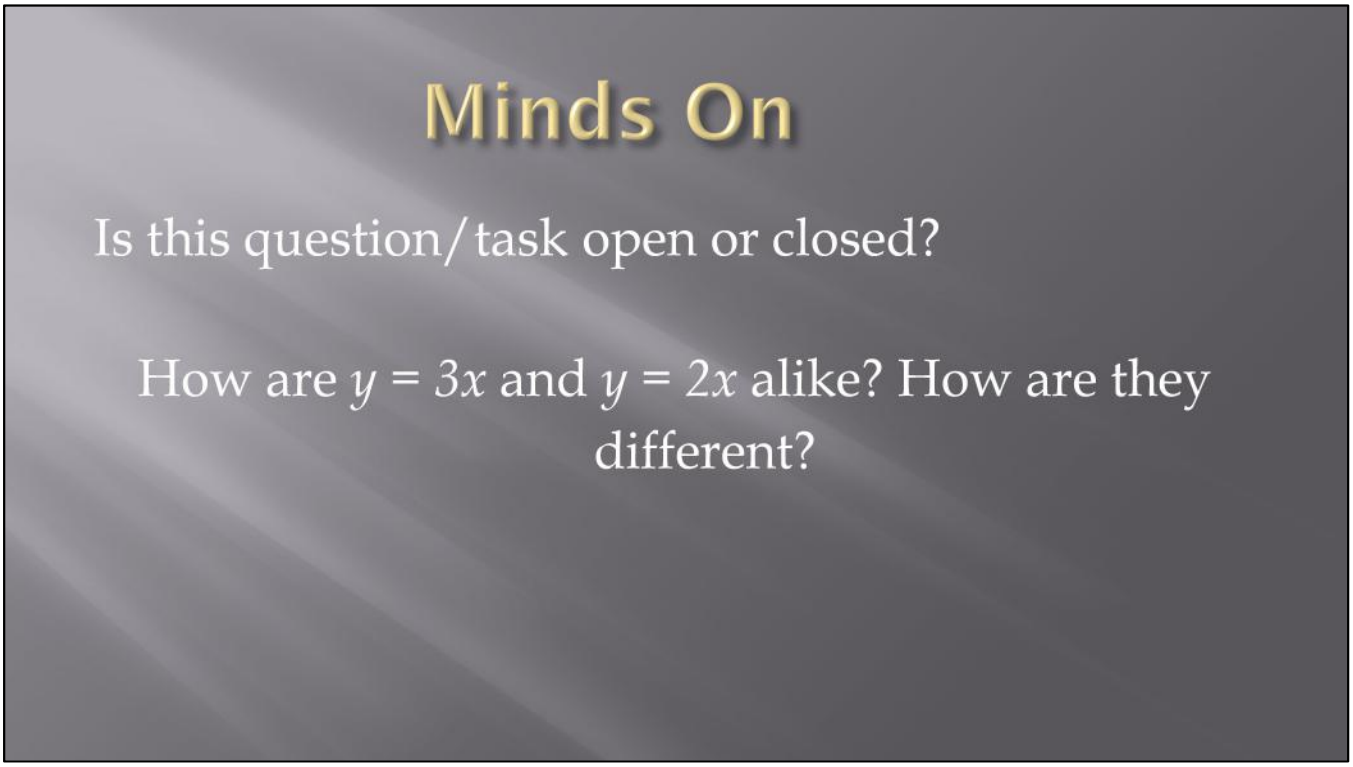
This is an open question. Closed questions are still valuable. There needs to be answers to some problems – and only one answer in many cases. This is usually the one we find in the back of the text – we hope!

Minds On

Is this question/task open or closed?

4. Find the equation of the line passing through (-3, -4) and (-1, 8). Express your equation in standard form!

This is a closed problem.



This is an open problem. Open problems allowed us to see where students were at an to allow all students access to the math at whatever level of understanding they were at. They also reinforced the idea that not all problems have an answer at the back of the book. In fact many real-world problems have an answer that they must find and determine if it is a reasonable solution based on their understanding.

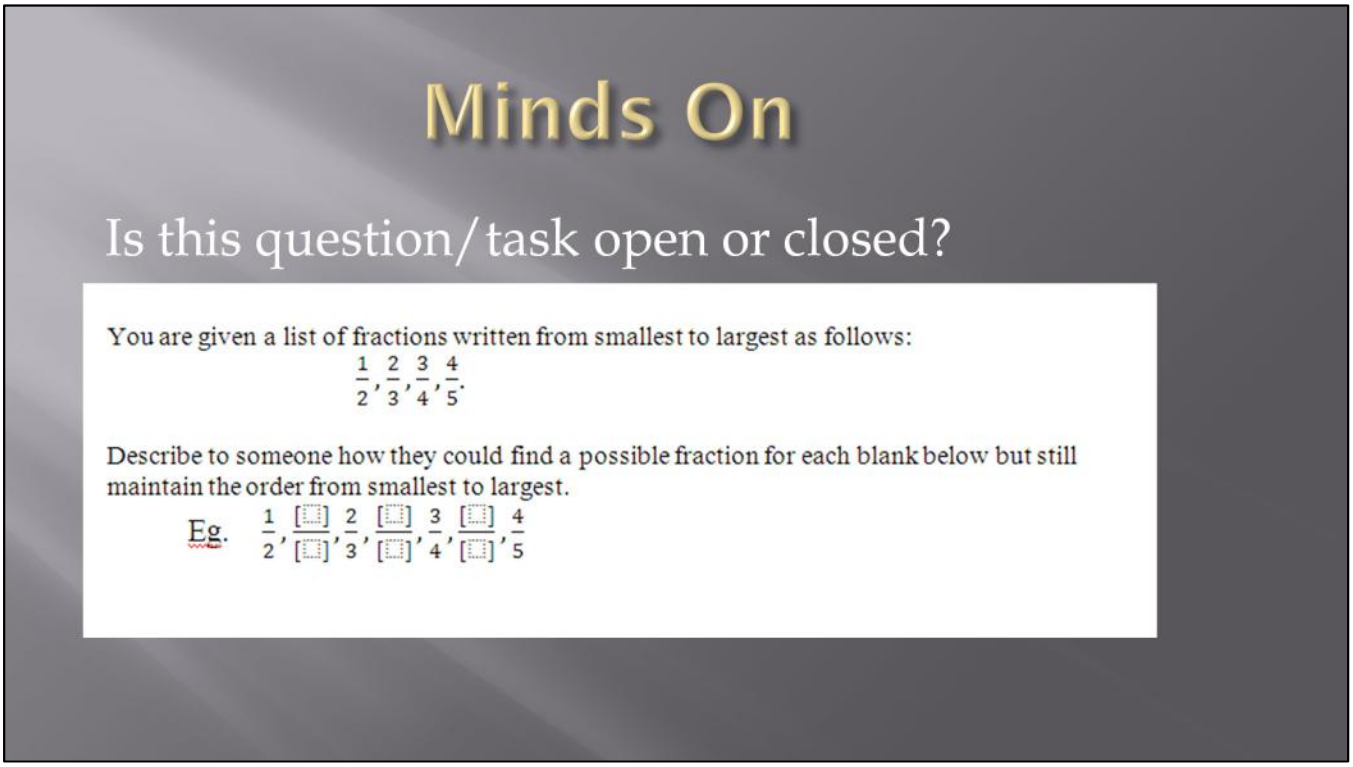
Minds On

Is this question/task open or closed?

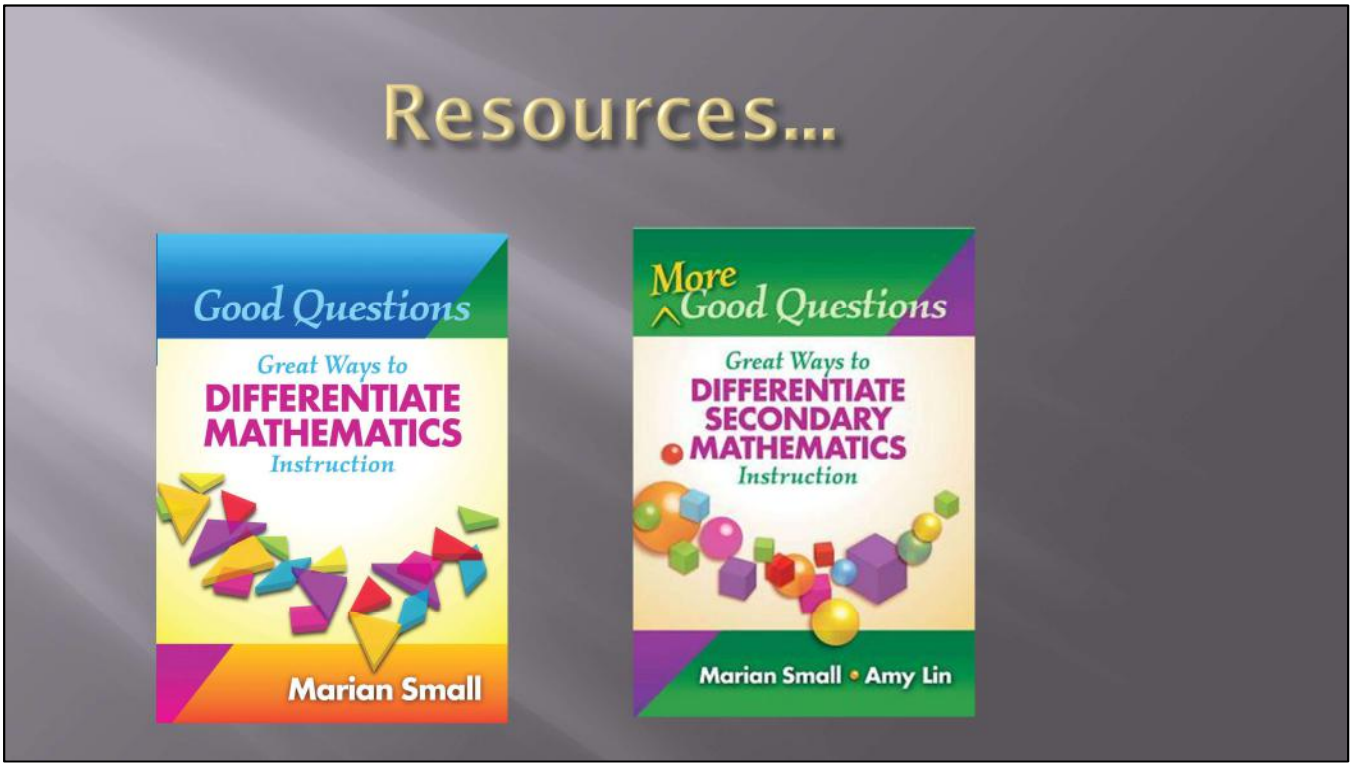
Write the following fractions in order from least to greatest.

$$\frac{3}{4}, \frac{2}{3}, \frac{1}{2}, \frac{4}{5}, \frac{1}{6}, \frac{7}{8}$$

This is a closed problem in that there is one answer but open in that there may be a variety of ways to solve it (open-routed).



This is an open problem (a variation on the preview problem). When we started using them in our instruction it stimulated discussion in the classroom. When we used them in our assessments, it allowed all students to engage with the problems. The issue of getting blank test papers came to an end. They also helped us develop good tasks for Academic students (esp. 11U and 12U courses) where the content made it challenging to find good problems/tasks like the ones we used in earlier grades and mentioned earlier. The math became a rich context to derive good problems/tasks.



Our guide in developing an approach to creating open problems – Marian Small and Amy Lin. Every teacher received a copy as a resource.

What does Small say?

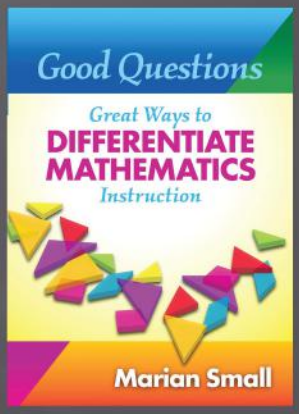
"A question is open when it is framed in such a way that a variety of responses or approaches are possible."

Open routed and/or open ended!

What strategies can we use to "open up" a question?

Common strategies…

- turning a question around...begin with the answer and ask for the question.
- asking for similarities and differences
- replacing a number with a blank
- asking for a number sentence
- add ambiguity… "soft" language (e.g. about, around, almost, etc)
- leave information out of the problem



Creating Open Questions

Here is a question that has appeared on a Grade 9 Applied assessment:

Cellphone Company A charges a flat fee of $20 plus $0.25 per text. Cellphone Company B charged a flat fee of $35 plus $0.15 per text. How many texts do I need to send so that the costs are equal between the two cellphone companies?

With your group, create an open question that addresses the same expectations as this question.

Creating Open Questions

Here is a question that has appeared on a Grade 9 Academic assessment:

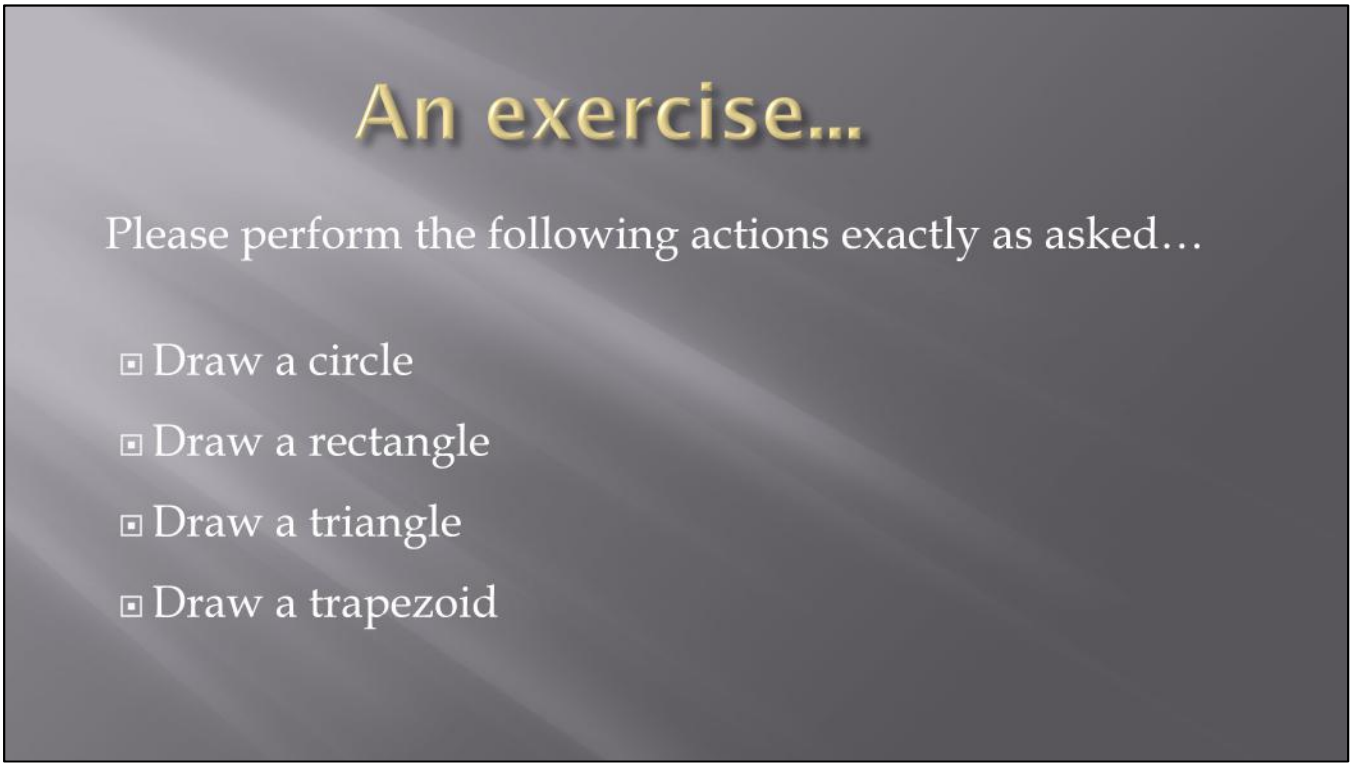
Determine the vertex for the parabola $y = x^2 + 8x + 15$.

With your group, create an open question that addresses the same expectations as this question.

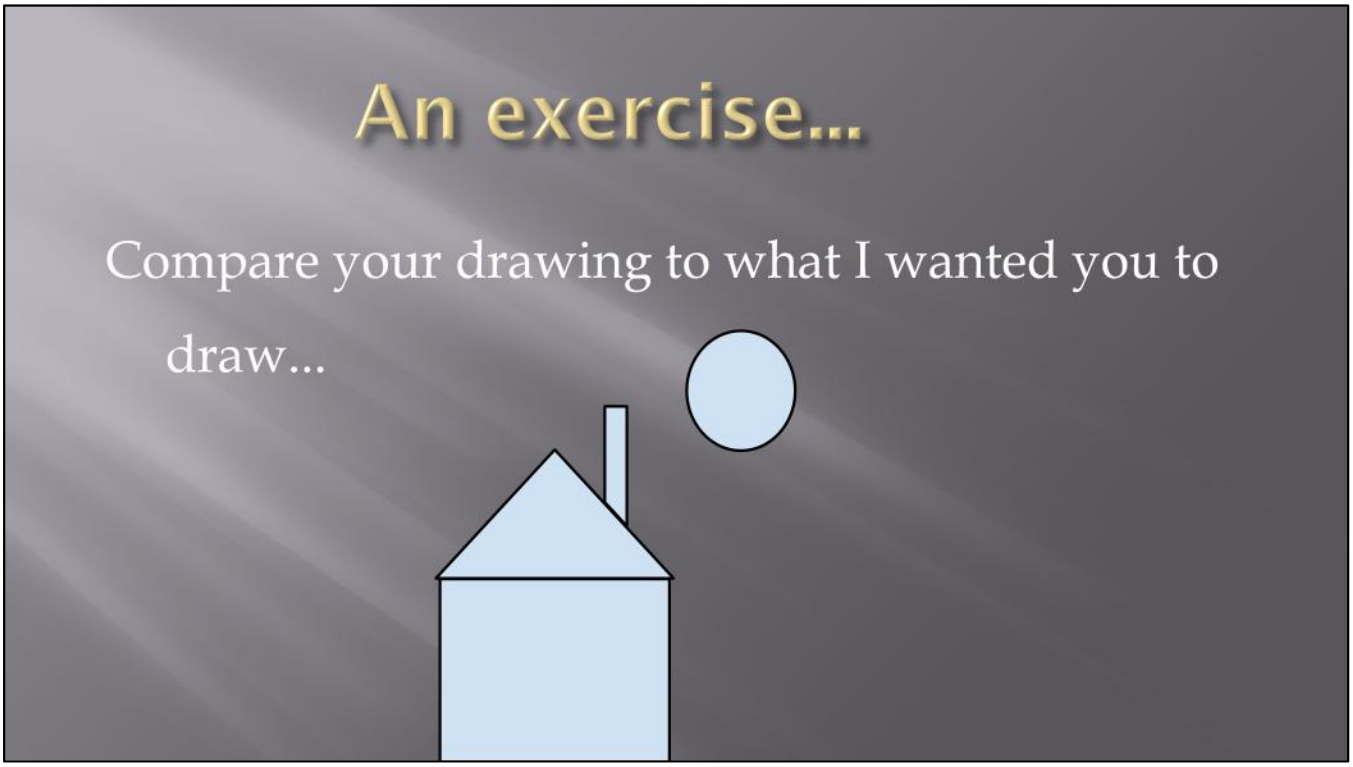
Getting students to adjust...

Thinking – The use of critical and creative thinking skills and/or processes*				
	The student:			
Use of planning skills – understanding the problem *(e.g., formulating and interpreting the problem, making conjectures)* – making a plan for solving the problem	uses planning skills with limited effectiveness	uses planning skills with some effectiveness	uses planning skills with considerable effectiveness	uses planning skills with a high degree of effectiveness
Use of processing skills – carrying out a plan *(e.g., collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions)* – looking back at the solution *(e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)*	uses processing skills with limited effectiveness	uses processing skills with some effectiveness	uses processing skills with considerable effectiveness	uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes *(e.g., problem solving, inquiry)*	uses critical/ creative thinking processes with limited effectiveness	uses critical/ creative thinking processes with some effectiveness	uses critical/ creative thinking processes with considerable effectiveness	uses critical/ creative thinking processes with a high degree of effectiveness

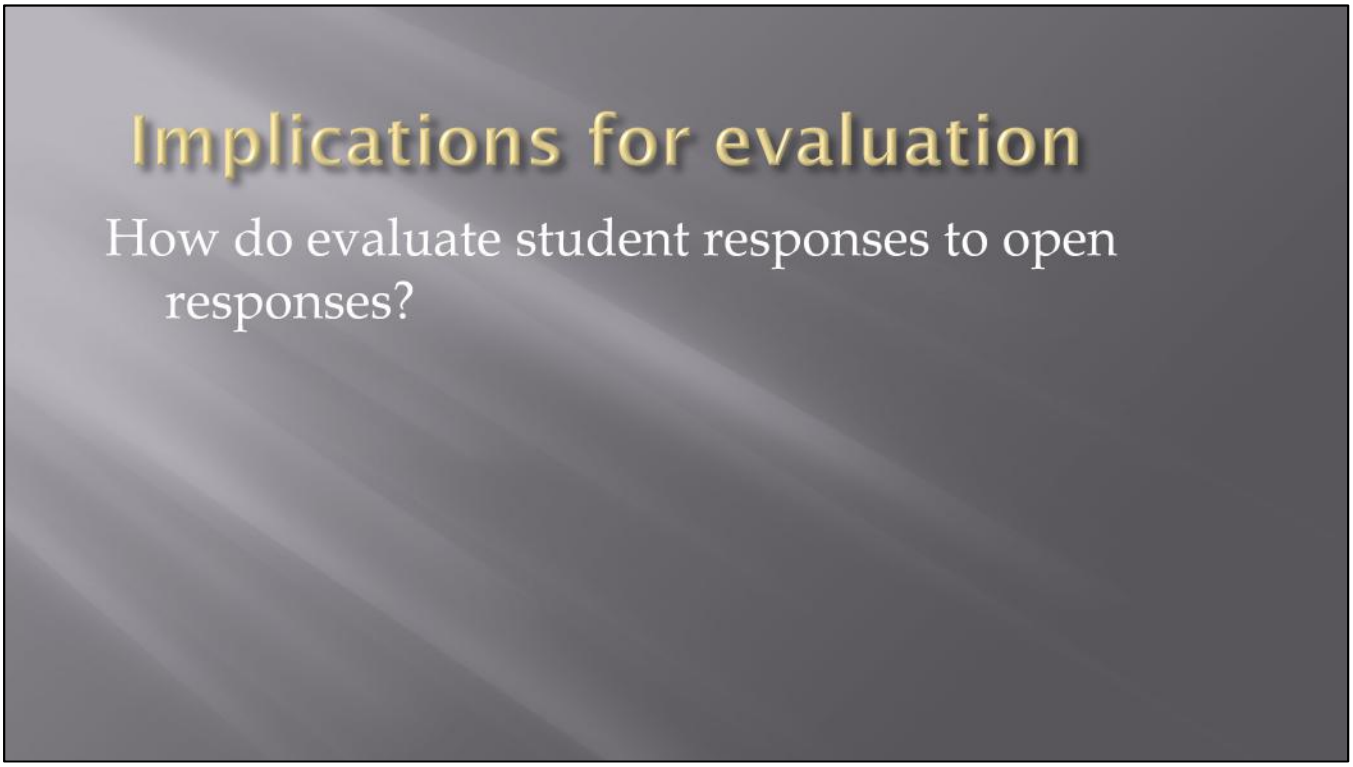
Incorporating open tasks in our teaching and assessments presented an unanticipated problem – we assumed students would be able to answer the problems. They would give us what we were looking for. Never ASSUME anything in teaching – especially math. Students needed to know the target in order to be able to hit it. We needed to provided explicit instruction on what it meant to solve a problem and what we were looking for. The AC category descriptors for Thinking helped us develop lessons to guide students.



An example from a Gr. 9 lesson early in the semester to help students clarify what it means to communicate their thinking.



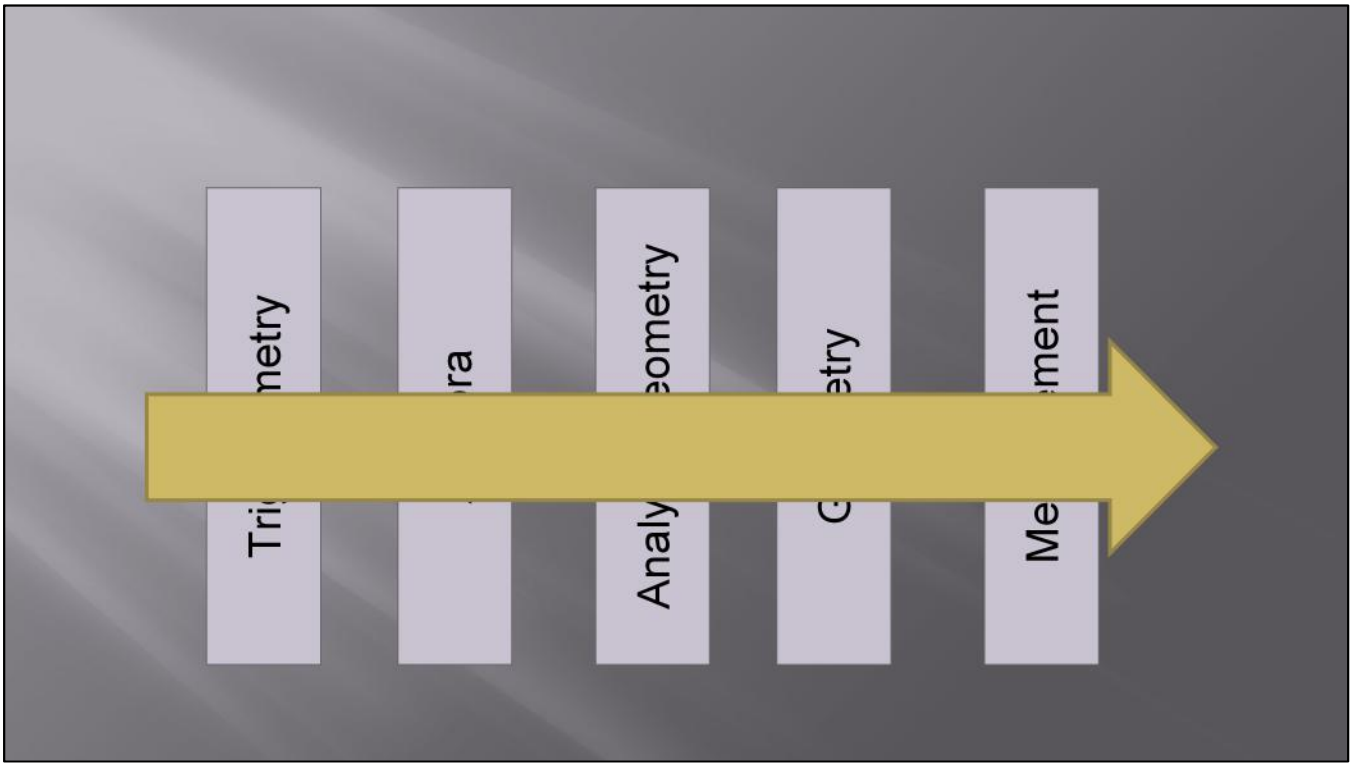
Did you come up with this picture? Why not? That was the point we were trying to make with our students.



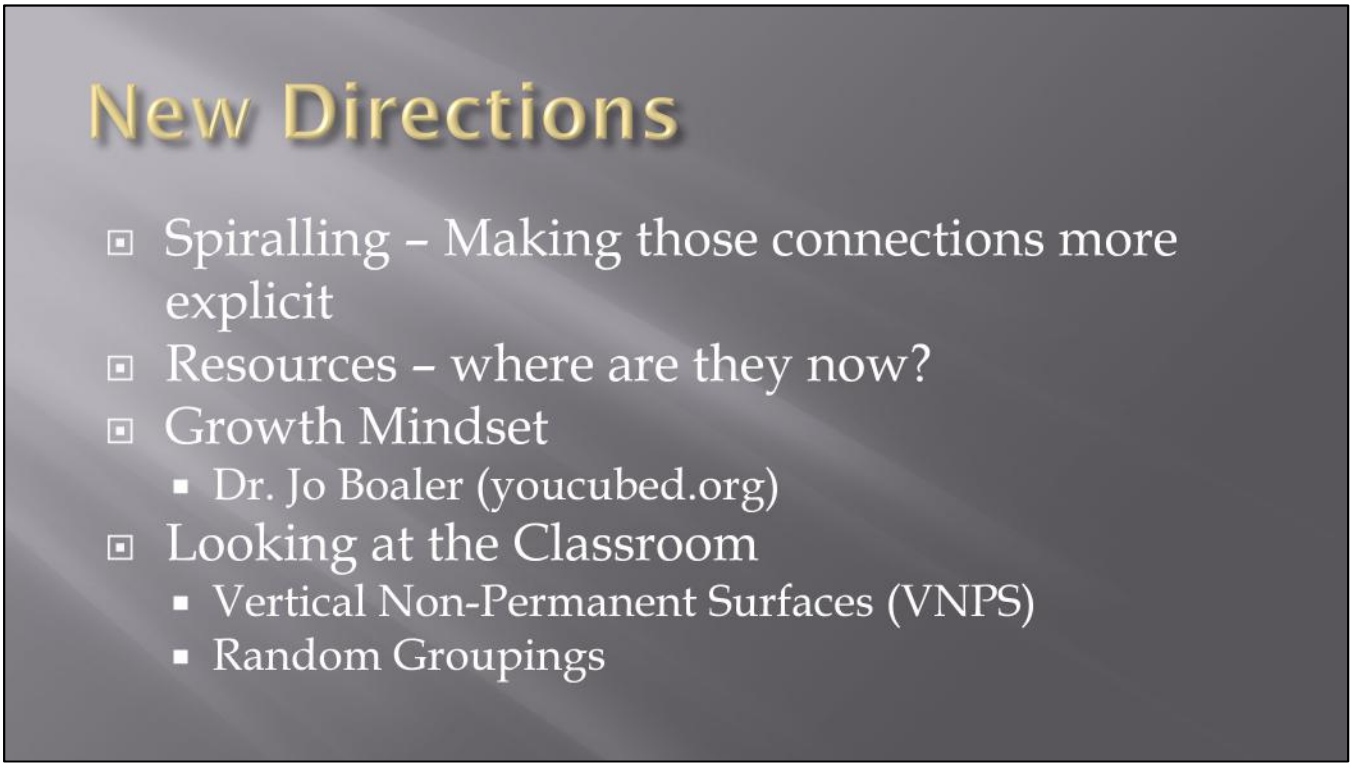
Evaluating student solutions to open tasks or any alternative form of evaluation is challenge. This required course teams to develop rubrics or checkbrics for each task and to sit down and look at some student solutions to clarify what they were looking for. A generic PS rubric was shared with students BEFORE the problem/task to allow for their own reflection and self-assessment.

The impact...

- improved student achievement and engagement
- MATH IS ABOUT SOLVING PROBLEMS!
- differentiation - allows ALL students an in
- helps foster a growth mindset - solving a problem is something ALL students can do
- values all solutions and all methods
- leads to rich discussion in class
- students reflect on performance and act on feedback



Embedding rich tasks and open problems allows for the blending of content in multiple strands so that students can make connections across strands.



What is happening in SOME math classrooms?
Spiralling within courses!
Resources – the math text is not what it used to be in the past. Teachers have access to more content (in a variety of media).
Growth Mindset – addressing perceptions around learning math that impede students and allow for persistence and perseverance.
If you want to see Dr. Boaler on Nov. 4, 2016 – visit www.oame.on.ca to register for the Leadership Conference.
VNPS and VRG – Based on work of Peter Liljedahl (http://www.peterliljedahl.com/)

Thanks!

- Thanks everyone for being such a great audience! I really enjoyed the dialogue and energy of the day.
- Loved playing the Game of Frogs with you!